UNIVERSITY OF STELLENBOSCH

DEPARTMENT OF MECHANICAL ENGINEERING

FINITE ELEMENT ANALYSIS OF A WING TYPE STRUCTURE WITH EXPERIMENTAL VERIFICATION OF RESULTS

PART A

THEORY AND COMPUTER PROGRAM

By

E.M.E. BAUMGARTNER

Promoted by

Mr. E. Fourie

Prof. R.J. du Preez

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TABLE OF CONTENTS

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A. THEORY

A1 INTRODUCTION

The advent of the high speed computer has revolutionized structural design in all spheres of Engineering. Up till then structural stress analysis was limited to over-simplification of the structure in question to comply with derived classical mathematical solutions.

In practice however the picture is very different, the structure usually being complex and highly redundant in nature. The techniques involving Energy Methods to solve such structures have been known for a long time. However they required weeks of hand calculations to solve only a small number of redundancies in a structure. Neville Shute mentions this in his book Slide Rule.

The development of Matrix algebra and the finite element method has made it possible to analyse, say, a complete aircraft structure in a matter of days, using a large capacity high speed computer.

Experimental results have shown that finite element stress analysis comes much closer to reality than the dated classical methods.

A2. THE FINITE ELEMENT METHOD

A2.1 Introduction

Let us look at a very simple example to start off with.

Consider a spring fastened at one end and a tensile force applied to its other end as in Figure 1.

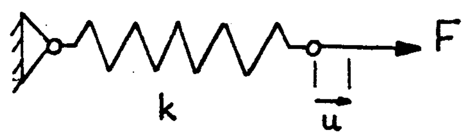


Figure 1

The spring will stretch a distance u and we have the well known spring equation

$$ku = F \qquad \text{.....................} \quad a2.1.1$$

If the situation were complicated slightly more and three springs were arranged as in Figure 2

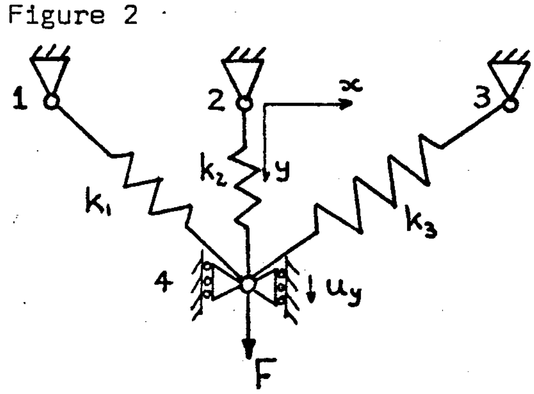


Figure 2

it would be necessary to determine the overall stiffness of the structure in the y-direction then apply the force to determine the deflection u_y.

$$\therefore \qquad (K_{1y} + K_{2y} + K_{3y}).u_y = F$$

If the roller supports were removed from the point 4 a deflection in the x-direction would also be experienced and the deflection u_y would not be the same as in the first case. The deflections u_x and u_y can now be split up into components parallel to the springs and the forces in the springs can be calculated.

This is in effect what the finite element method does.

A structure can be split up into elements e.g. Rods, panels, plates and beams all connected at their ends or corners. These connection points are called nodal points.

Each element has a certain stiffness which can be expressed, in terms of its geometry and elastic properties, at each of its nodal points.

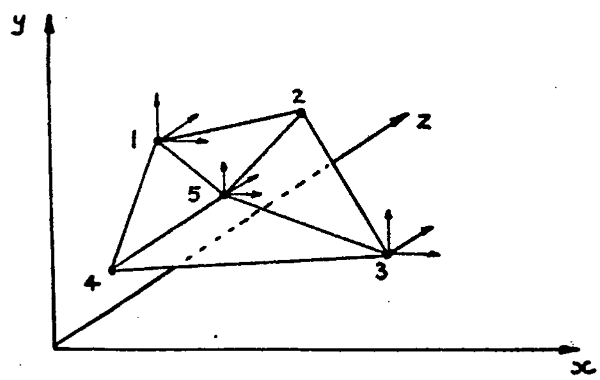


Figure 3

In the structure shown in Figure 3 there are 5 nodal points. Each element has a certain stiffness in each of the 3 global axes directions and the sum of the stiffnesses, in a specific direction at a nodal point, then represents the stiffness of that nodal point in that direction. In Figure 3 there are thus 5 x 3 unknown deflections i.e. 15 simultaneous linear equations are needed to solve for the unknowns.

Written in matrix form the equations for the entire structure looks similar to equation a2.1.1

$$\underset{\sim\sim}{KU} = \underset{\sim}{P} \qquad \text{.................... a2.1.2}$$

Where $\underset{\sim}{K}$ is a square symmetric matrix of nodal point stiffness coeffi= cients, $\underset{\sim}{U}$ is a column vector of the unknown displacements and $\underset{\sim}{P}$ is the column vector of the nodal point loads.

The set of equations a2.1.2 are solved by inversion of the $\underset{\sim}{K}$ matrix

$$\underset{\sim}{U} = \underset{\sim}{K}^{-1} \underset{\sim}{P} \qquad \text{....................} \quad \text{a2.1.3}$$

and then the solved deflections back substituted into the stiffness matrices of the individual elements to determine the stresses and strains in them.

This method is known as the displacement stiffness method, and is now widely used in many large finite element programs such as ASKA and NASTRAN.

The reciprical of the displacement method is the force or flexibility method in which the forces are the unknowns. This method is more compli= cated to apply and is thus not as popular as the displacement method although hybrid elements incorporating both the methods are in use.

A2.2 Formulation of the Displacement Method

The theory of finite elements can be extended to incorporate special material properties such as non-isotropy and non-linearity, however this discussion will be based on the assumption that the materials used have a linear elastic region. This assumption is made in most books on Strength of Materials and Elasticity Theory.

A2.2.1 Basic elasticity equations

It can be shown (Ref. 6) that if a body is deformed in such a way so that the deformation in the x, y and z directions can be expressed as functions u, v, and w respectively, then the strains can be expressed as follows in the kinematic equations.

$$\left.\begin{aligned}\varepsilon_{xx} &= \frac{\partial u}{\partial x}\\ \varepsilon_{yy} &= \frac{\partial v}{\partial y}\\ \varepsilon_{zz} &= \frac{\partial w}{\partial z}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.2.1$$

$$\left.\begin{aligned}\varepsilon_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\\ \varepsilon_{xz} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\\ \varepsilon_{yz} &= \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.2.2$$

In the one-dimensional case the stress and the strain are related as follows

$$\varepsilon = \frac{1}{E} \cdot \sigma \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.2.3$$

where E is the Modulus of Elasticity and σ the stress in the direction of the chosen strain. Poisson discovered that in the three-dimensional strain case the relationship is as follows.

$$\left.\begin{aligned}\varepsilon_{xx} &= \frac{1}{E}\left[\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})\right]\\ \varepsilon_{yy} &= \frac{1}{E}\left[\sigma_{yy} - \nu(\sigma_{xx} + \sigma_{zz})\right]\\ \varepsilon_{zz} &= \frac{1}{E}\left[\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})\right]\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad a2.2.4$$

where ν is called Poisson's ratio.

For shear strain we have

$$\left.\begin{aligned} \varepsilon_{xy} &= \frac{\sigma_{xy}}{G} \\ \varepsilon_{xz} &= \frac{\sigma_{xz}}{G} \\ \varepsilon_{yz} &= \frac{\sigma_{yz}}{G} \end{aligned}\right\} \qquad \text{.................... a2.2.5}$$

where G is called the Modulus of Elasticity in Shear and is found to be related to E as follows

$$G = \frac{E}{2(1 + \nu)} \qquad \text{.................... a2.2.6}$$

"Special" cases of the general theory are frequently encountered, such as Plane Stress which occurs approximately in thin sheet panels and which obviously has application in an aircraft structure.

To derive the plane stress equations the terms σ_{zz} σ_{xz} and σ_{yz} are set to zero and the state of stress is then specified by σ_x, σ_y and σ_{xy} only. After manipulation equations a2.2.4 and a2.2.5 can be written

$$\left.\begin{aligned} \sigma_{xx} &= \frac{E}{1 - \nu^2}\,\varepsilon_{xx} + \frac{\nu E}{1 - \nu^2}\,\varepsilon_{yy} \\ \sigma_{yy} &= \frac{\nu E}{1 - \nu^2}\,\varepsilon_{xx} + \frac{E}{1 - \nu^2}\,\varepsilon_{yy} \\ \sigma_{xy} &= \frac{E}{2(1 + \nu)}\,\varepsilon_{xy} \end{aligned}\right\} \qquad \text{.............. a2.2.7}$$

which can be expressed in matrix form as follows

$$\bar{\sigma} = \bar{E}\,\bar{\varepsilon}$$

or

$$\underbrace{\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}}_{\bar{\sigma}} = \underbrace{\frac{E}{1 - \nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}}_{\bar{E}} \underbrace{\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{bmatrix}}_{\bar{\varepsilon}} \qquad \text{.............. a2.2.8}$$

A2.2.2 Work and Energy

Work is defined as a force multiplied by the distance through which it moves,

$$W = \vec{F} \cdot \vec{s} \qquad \text{.....................} \quad a2.2.9$$

or in differential form

$$dW = \vec{F} \, d\vec{s} \qquad \text{.....................} \quad a2.2.10$$

Thus if a linearly elastic system (e.g. a structure) undergoes a deflec= tion du then the work done on the system is

$$dW = P \, du \qquad \text{.....................} \quad a2.2.11$$

P is the applied load.

The work done by the structure in resisting the applied forces is

$$dU = \underline{F} \cdot du \qquad \text{.....................} \quad a2.2.12$$

where F are the internal forces of the system (material Stresses)

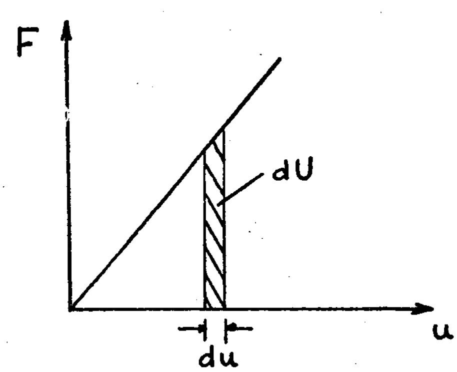


Figure 4

At this stage the principle of virtual or imaginary displacements is introduced which states that:

For a system which is in equilibrium, the sum of the internal and external work done during a virtual displacement is equal to zero, on condition that the kinematic equations and the geometric boundary conditions are satisfied.

Mathematically the virtual displacement δu behaves the same as the differential quantity du.

For virtual displacements the equations of work now become

$$\delta W = P\,\delta u \qquad \text{a2.2.13}$$

$$\delta U = F.\delta u \qquad \text{a2.2.14}$$

and from the above stated Principle of Virtual Displacements

$$\delta U + \delta W = 0 \qquad \text{a2.2.15}$$

The next step is to relate the internal forces to the external ones by means of the structures elastic properties.

Consider now the well known linear stress-strain diagram of a material Figure 5.

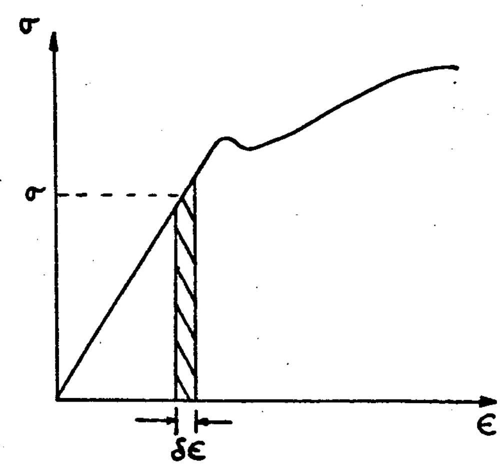


Figure 5

The expression $\sigma\delta\varepsilon$ has as its dimensions $\frac{N}{m^2} \cdot \frac{m}{m}$ which is Work/unit volume. Thus if the integral is taken over the volume of the system, or a single element, the variation of the strain energy of the system can be found.

$$\delta U = \int_V \bar{\sigma}\, \delta\bar{\varepsilon}\, dV \qquad \text{a2.2.16}$$

The system is acted upon by external forces and body forces (gravitational, initial strains, temperature effects) and these are also given per unit area (external) and per unit volume (body) and integrated over the area of application and volume of the system respectively.

$$\delta W = \int_V \bar{g}\ \delta\bar{u}\ dV + \int_A \bar{q}\ \delta\bar{u}\ dA \qquad \text{.............. a2.2.17}$$

Substituting a2.2.16 and a2.2.17 into a2.2.15 gives

$$- \int_V \bar{\sigma}\ \delta\bar{\varepsilon}\ dV + \int_V \bar{g}\ \delta\bar{u}\ dV + \int_A \bar{q}\ \delta\bar{u}\ dA = 0\ . \qquad \text{a2.2.18}$$

The minus sign in front of the δU term arises from the fact that the stresses oppose the applied forces. Equation a2.2.18 is subject to the conditions that the kinematic equations and the prescribed boundary con= ditions are satisfied.

As was seen previously, a structure is divided into a finite number of elements, connected at their nodal points, the next step thus, is to establish functions for the displacements in the x, y and z directions and express the displacements within the system in terms of the nodal point deflections which arise out of the solution of equation a2.1.3.

To obtain the strains $\bar{\varepsilon}$ in the system it follows that the chosen displacement functions must be continuous differentiable at least once. If they still satisfy the geometric boundary conditions then they are allowable functions (Ref. 4).

It can be shown that if the displacement functions

1. include all possible rigid body displacements

2. include uniform strain conditions i.e. constant stress conditions

3. satisfy the geometric interelement and boundary conditions

Then the finite element method converges monotonously to the correct solution as the number of elements is increased.

If conditions 1. and 2. only are satisfied then convergence still takes place but not monotonously.

It has been seen from equations a2.2.1 and a2.2.2 that the displacement functions have to be partially differentiated in one form or another to produce the strains. These partial differentiations will be signified by the comma.

$$\therefore \qquad \bar{\varepsilon} = \bar{u},$$

where $\bar{\varepsilon}$ takes into account all the strains in the system and $\bar{u}$ the displacements.

$$\therefore \qquad -\int_V \bar{\sigma}\, \delta\bar{u},\, dV + \int_V \bar{g}\, \delta\bar{u}\, dV + \int_A \bar{q}\, \delta\bar{u}\, dA = 0 \quad \ldots\ldots \quad \text{a2.2.19}$$

Let the displacement functions be

$$\bar{u}(x,\ y,\ z) = \Phi(x,\ y,\ z)\,\hat{\bar{u}}$$

$$\left.\begin{aligned} \therefore \qquad \delta\bar{u} &= \Phi\delta\hat{\bar{u}} \\ \therefore \qquad \delta\bar{u}, &= \Phi,\ \delta\hat{\bar{u}} \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{a2.2.20}$$

where $\hat{\bar{u}}$ are the nodal point deflections. Substituting a2.2.20 into a2.2.19

$$\delta\hat{\bar{u}}\,\{-\int_V \Phi,\ \bar{\sigma}\, dV + \int_V \bar{g}\,\Phi\, dV + \int_A \Phi\,\bar{q}\, dA\} = 0$$

$$\therefore \quad -\int_V \Phi,\ \bar{\sigma}\, dV + \int_V \bar{g}\,\Phi\, dV + \int_A \Phi\,\bar{q}\, dA = 0 \quad \ldots \quad \text{a2.2.21}$$

From equation a2.2.8 it can be seen that

$$\bar{\sigma} = \bar{E}\,\bar{\varepsilon}$$

also

$$\bar{\varepsilon} = \bar{u},$$

$$= \Phi,\ \hat{\bar{u}}$$

$$\therefore \quad \bar{\sigma} = \bar{E}\,\Phi,\ \hat{\bar{u}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{a2.2.22}$$

Substituting into a2.2.21

$$- \int_V \Phi, \overline{E}\, \Phi, \hat{\overline{u}}\, dV + \int_V \overline{g}\, \Phi\, dV + \int_A \overline{q}\, \Phi\, dA = 0$$

But the terms $\Phi,$, $\overline{E}$, $\hat{\overline{u}}$, Φ are matrices, thus from the laws of matrix algebra where

$$(A)^2 = A^T A$$

∴ we get

$$- \int_V \overline{\Phi},^T \overline{E}\, \overline{\Phi}, \hat{\overline{u}}\, dV + \int_V \overline{\Phi}^T \overline{g}\, dV + \int_A \overline{\Phi}^T \overline{q}\, dA = 0 \quad .. \quad a2.2.23$$

From the first term in equation a2.2.23 must come a stiffness multiplied by a deflection thus

$\int_V \overline{\Phi},^T \overline{E}\, \overline{\Phi},\, dV$ is the stiffness matrix of the system and

$\hat{\overline{u}}$ the nodal point deflections. The stiffness matrix is signified by the symbol $\overline{k}$ for an element and $\overline{K}$ for the complete structure.

The last two terms of the equation represent the loads on the system. The $\overline{g}$ vector can incorporate initial strains due to lack of fit, tempe= rature strains and gravitational forces due to the system's own weight. The $\overline{q}$ vector represents the external loads on the system. Now as men= tioned previously the system is divided up into a finite number of elements connected at a finite number of nodal points thus the external and internal forces must be applied at the nodal points. If point loads are applied to the system then there should be no problem, the system can be split up in such a way that the point load coincides with a nodal point. If however the system is loaded with a distributed load then equivalent nodal point forces must be determined using the integral expression in equation a2.2.23.

Thus taking the force terms to the RHS of the equation and combining them gives for the whole structure.

$$\bar{K}\,\hat{\bar{U}} = \bar{P} \qquad \text{a2.2.24}$$

which is the same as the original simple spring equation a2.1.1 and for a single element

$$\bar{k}\,\hat{\bar{u}} = \bar{p} \qquad \text{a2.2.25}$$

A2.3 Some Common Element Stiffness Matrices

A2.3.1 Endload or Rod Element

This is the simplest element used in finite element analysis. It is generally used in the modeling of a structure where a component is expected to be in pure tension or compression i.e. it has no bending stiffness. It is used to model pin jointed truss structures, beam flanges and stiffeners in aircraft structures.

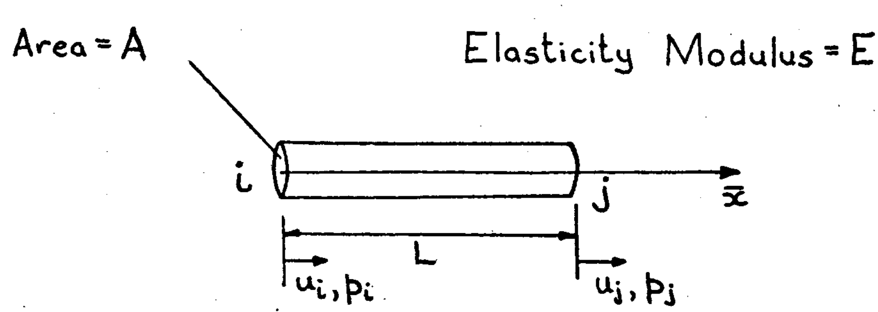


Figure 6

Figure 6 shows the rod element lying along the $\bar{x}$-axis with its nodal point deflections and loads given by u_i, u_j and p_i, p_j respectively.

For a uniform rod the strain is constant, therefore we can assume that the displacement function u is a linear function of x

$$\bar{u} = \Phi\,(x)$$

$$\bar{u} = A_1 x + A_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.3.1$$

$$\therefore \quad \varepsilon_{\bar{x}\bar{x}} = \frac{\partial \bar{u}}{\partial x} = A_1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.3.2$$

For the rod in question the following boundary conditions exist

at $\quad x = 0$

$\quad u = u_i$

$\therefore \quad A_2 = u_i$

and at $\quad x = L$

$\quad u = u_j$

$\quad u_j = A_1 L + u_i$

$\therefore \quad (u_j - u_i)/L = A_1$

$$\therefore \quad \bar{u} = \frac{(u_j - u_i)}{L} x + u_i \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.3.3$$

$$= u_i \left(1 - \frac{x}{L}\right) + \frac{u_j}{L} x$$

$$= \begin{bmatrix} (1 - \frac{x}{L}) & \frac{x}{L} \end{bmatrix} \begin{bmatrix} u_i \\ u_j \end{bmatrix}$$

$$\bar{u} = \bar{\Phi} \, \hat{\bar{u}}$$

$$\therefore \quad \bar{u}, = \bar{\Phi}, \, \hat{\bar{u}}$$

$$= \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{bmatrix} u_i \\ u_j \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.3.4$$

From equation a2.2.23 the stiffness matrix is

$$\bar{k} = \int_V \bar{\Phi},^T \bar{E} \, \bar{\Phi}, \, dV \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.3.5$$

For the one dimensional case

$$\bar{E} = E$$

$$\therefore \qquad \bar{k} = E\int_V \begin{bmatrix} -\frac{1}{L} \\ \frac{1}{L} \end{bmatrix} \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} dV$$

$$\therefore \qquad \bar{k} = E\int_V \begin{bmatrix} \frac{1}{L^2} & -\frac{1}{L^2} \\ -\frac{1}{L^2} & \frac{1}{L^2} \end{bmatrix} dV$$

and if the area is constant

$$\bar{k} = EA\int_0^L \begin{bmatrix} \frac{1}{L^2} & -\frac{1}{L^2} \\ -\frac{1}{L^2} & \frac{1}{L^2} \end{bmatrix} dx$$

$$\bar{k} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \qquad \text{.....................} \quad \text{a2.3.6}$$

which is the stiffness matrix for the rod element in local co-ordinates.

It is customary when analysing a structure to analyse it according to some reference set of axes. Let's call them the global axes. Now not all the elements in the structure are going to be orientated such that they lie in the planes of the global axes so the element has to be derived in its own set of axes,called local axes,and then transformed in such a way that its stiffnesses are then in the planes of the global axes. The deflections will then be in the directions of the global axes.

The transformation matrices will be dealt with in the next section.

A2.3.2 The Plane Stress Triangle

This element is more commonly known as the C.S.T. element (Constant

Strain Triangle) because its displacement functions are chosen such that the strains ε_{xx}, ε_{yy} and ε_{xy} are constant throughout the element.

Figure 7 shows the element in its local co-ordinate system.

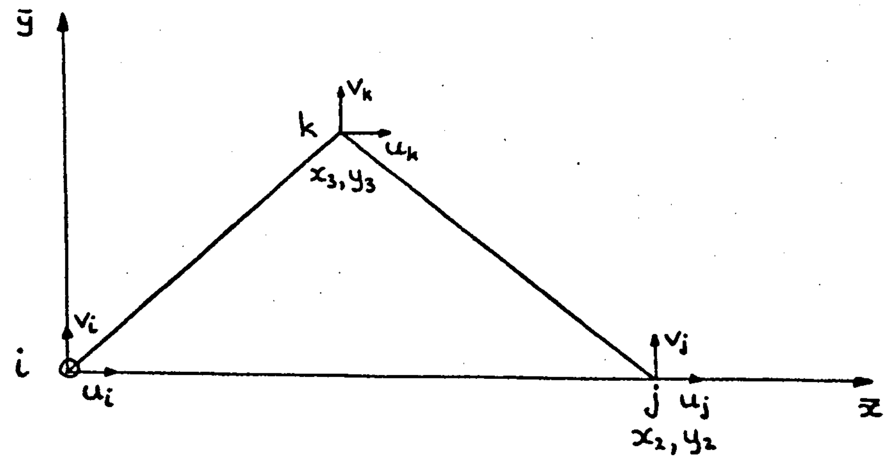


Figure 7

It can be seen that if the displacement functions are given by

$$\left.\begin{aligned} u &= A_1 x + A_2 y + A_3 \\ &\text{and} \\ v &= B_1 y + B_2 x + B_3 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad a2.3.7$$

then
$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = A_1$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = B_1$$

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = A_2 + B_2$$

which gives the constant strains required.

Now by substituting the three boundary conditions into equations a2.3.7 i.e.

$$u(0,0) = u_i \qquad u(x_2,0) = u_j \qquad u(x_3, y_3) = u_k$$
$$v(0,0) = v_i \qquad v(x_2,0) = v_j \qquad v(x_3, y_3) = v_k$$

gives

$$A_1 = (u_j - u_i)/x_2$$

$$A_2 = \left[u_k\, x_2 + (x_3 - x_2)\, u_i - u_j\, x_3\right]/x_2\, y_3$$

$$A_3 = u_i$$

$$B_1 = \left[v_k\, x_2 + (x_3 - x_2)\, v_i - v_j\, x_3\right]/x_2\, y_3$$

$$B_2 = (v_j - v_i)/x_2$$

$$B_3 = v_i$$

$$u = (u_j - u_i)y_3/x_2\, y_3 .\; x + \left[u_k\, x_2 + (x_3 - x_2)\, u_i - u_j\, x_3\right]/x_2\, y_3 .\; y + u_i . \frac{x_2\, y_3}{x_2\, y_3}$$

We can see that

$$\text{Area}_\Delta = \tfrac{1}{2}\, x_2\, y_3 = \Delta$$

$$\therefore \quad u = \frac{1}{2\Delta}\left[\{-y_3\, x + (x_3 - x_2)\, y + x_2\, y_3\}\, u_i + (y_3\, x - x_3\, y)\, u_j + x_2\, y\, u_k\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{a2.3.8}$$

$$\therefore \quad v = \left[v_k\, x_2 + (x_3 - x_2)\, v_i - v_j\, x_3\right]/x_2\, y_3 .\; y + (v_j - v_i)y_3/x_2\, y_3 .\; x + \frac{x_2\, y_3}{x_2\, y_3}\, v_i$$

$$= \frac{1}{2\Delta}\left\{\left[-y_3\, x + (x_3 - x_2)\, y + x_2\, y_3\right] v_i + (y_3\, x - x_3\, y)\, v_j + x_2\, y\, v_k\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{a2.3.9}$$

From equations a2.3.8/9 we can write

$$\begin{bmatrix} u \\ v \end{bmatrix} = \frac{1}{2\Delta}\begin{bmatrix} N_1 & 0 & N_2 & 0 & N_3 & 0 \\ 0 & N_4 & 0 & N_5 & 0 & N_6 \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{bmatrix}$$

$$(\bar{u}) = (\bar{\Phi})\, (\hat{\bar{u}})$$

Where N_i $i = 1\text{-}6$ are functions of x and y.

$$\therefore \quad (\bar{u},) = \frac{1}{2\Delta}\begin{bmatrix} N_{1,x} & 0 & N_{2,x} & 0 & N_{3,x} & 0 \\ 0 & N_{4,y} & 0 & N_{5,y} & 0 & N_{6,y} \\ N_{1,y} & N_{4,x} & N_{2,y} & N_{5,x} & N_{3,y} & N_{6,x} \end{bmatrix} \begin{bmatrix} u_i \\ v_i \\ u_j \\ v_j \\ u_k \\ v_k \end{bmatrix}$$

$$= \frac{1}{2\Delta}\begin{bmatrix} -y_3 & 0 & y_3 & 0 & 0 & 0 \\ 0 & (x_3-x_2) & 0 & -x_3 & 0 & 0 \\ (x_3-x_2) & -y_3 & -x_3 & y_3 & x_2 & 0 \end{bmatrix} (\hat{\bar{u}})$$

$$= (\bar{\Phi},)\ (\hat{\bar{u}})$$

$$\therefore \quad \bar{k} = \int_V \bar{\Phi},^T\ \bar{E}\ \bar{\Phi},\ dV$$

where $\bar{E}$ is the matrix given in equation a2.2.8. It can be seen however that the terms in $\bar{\Phi},$ are all constants, and if the thickness of the element is constant then

$$\bar{k} = \Delta . \bar{\Phi},^T\ \bar{E}\ \bar{\Phi},\ .\ t$$

which is the stiffness matrix for the plane stress C.S.T. element in local co-ordinates.

A2.3.3 Quadrilateral Elements

There are various methods used in assembling a four sided plane stress element.

Firstly it can be built up out of four C.S.T. elements and the internal nodal point can be eliminated from the set of equations

$$\bar{k}\ \hat{\bar{u}} = \bar{p}$$

by setting the $\bar{x}$ and $\bar{y}$ forces at the internal point equal to zero. A computer condensation routine is given in Ref. 3.

Secondly displacement fields for the quadrilateral can be found and the normal procedure followed, this however is a long and complex business and the isoparametric formulation has become necessary for speedy assem= bly of the stiffness matrix. This is explained fully in Refs. 2, 3 and 4. A computer routine for the assembly of the plane stress quadrilateral stiffness matrix, using isoparametric formulation can be found in Ref. 3.

A2.3.4 Approximate elements

Shear Triangle

For those who are familiar with the classical analysis of aircraft struc= tures it will be remembered that the structure is usually idealized into a series of booms and shear carrying panel members.

The boom is the normal endload element derived in section A2.3.1. The triangular shear shear panel can be derived in the same way as the C.S.T. element of section A2.3.2 but the $\overline{E}$ matrix becomes

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G \end{bmatrix}$$

where $G = \dfrac{E}{2(1+\nu)}$

Multiplying out

$$\overline{k} = \int_V \overline{\Phi},^T \overline{E}\, \overline{\Phi}, \, dV$$

$$= \frac{Gt}{4}\begin{bmatrix} (x_3-x_2)^2 & & & & & \\ -y_3(x_3-x_2) & y_3^2 & & \text{sym} & & \\ -x_3(x_3-x_2) & x_3y_3 & x_3^2 & & & \\ y_3(x_3-x_2) & -y_3^2 & -x_3y_3 & y_3^2 & & \\ x_2(x_3-x_2) & -x_2y_3 & -x_2x_3 & x_2y_3 & x_2^2 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

It can be seen that this element has no stiffness in the v_k direction, this is overcome in the structure however because each side of the panel will be stiffened by a boom.

Quadrilateral Shear Panel (Ref. 1)

The assumption is made that the shear strain is constant throughout the panel thus

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = C_1 \qquad \text{a2.3.10}$$

and

$$\left.\begin{aligned} \varepsilon_{xx} &= \frac{\partial u}{\partial x} = C_2 + C_3\, y \\ \varepsilon_{yy} &= \frac{\partial v}{\partial y} = C_4 + C_5\, x \end{aligned}\right\} \qquad \text{a2.3.11}$$

Integration of equation a2.3.11 yields

$$\left.\begin{aligned} \mu &= C_2\, x + C_3\, xy + F_1(y) \\ v &= C_4\, y + C_5\, xy + F_2(x) \end{aligned}\right\} \qquad \text{a2.3.12}$$

Substituting these into a2.3.10 yields

$$C_3\, x \frac{\partial F_2}{\partial x} + C_5\, y + \frac{\partial F_1}{\partial y} = C_1$$

or

$$G_1\,(x) + G_2\,(y) = C_1$$

Thus $G_1\,(x)$ and $G_2\,(y)$ must be constants

$$\therefore \qquad \frac{\partial F_1}{\partial y} + C_5\, y = C_6$$

$$\frac{\partial F_2}{\partial x} + C_3\, x = C_9$$

$\therefore$ Integrating yields

$$\left.\begin{aligned} F_1 &= -\frac{C_5}{2} y^2 + C_6 y + C_7 \\ F_2 &= -\frac{C_3}{2} x^2 + C_9 x + C_8 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.3.13$$

Substituting a2.3.13 into a2.3.12 yields

$$\left.\begin{aligned} u &= C_2 x + C_3 xy + C_6 y + C_7 - C_5 y^2/2 \\ v &= C_4 y + C_5 xy + C_9 x + C_8 - C_3 x^2/2 \\ \text{and} \quad C_9 &= C_1 - C_6 \end{aligned}\right\} \qquad \ldots\ldots\ldots\ldots \quad a2.3.14$$

By substituting the boundary conditions, at the four nodal points, into equations we get the relationship

$$\bar{u} = \bar{\Phi}\,\hat{\bar{u}}$$

and from $\bar{k}\int_V \bar{\Phi}^T, \bar{E}\,\bar{\Phi}, dV$ with $\bar{E} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & G \end{bmatrix}$

yields (Ref. 1)

$$\bar{k} = \frac{Gt}{2A} \begin{bmatrix} x^2_{nj} & & & & & & & \\ -y_{ni}\,x_{nj} & y^2_{ni} & & & & \text{sym} & & \\ -x_{mi}\,x_{nj} & y_{ni}\,x_{mi} & x^2_{mi} & & & & & \\ y_{mi}\,x_{nj} & -y_{mi}\,y_{ni} & -y_{mi}\,x_{mi} & y^2_{mi} & & & & \\ -x^2_{nj} & y_{ni}\,x_{nj} & x_{mi}\,x_{nj} & -y_{mi}\,x_{nj} & x^2_{nj} & & & \\ y_{ni}\,x_{nj} & -y^2_{ni} & -y_{ni}\,x_{mi} & y_{mi}\,y_{ni} & -y_{ni}\,x_{nj} & y^2_{ni} & & \\ x_{mi}\,x_{nj} & -y_{ni}\,x_{mi} & -x^2_{mi} & y_{mi}\,x_{mi} & -x_{mi}\,x_{nj} & y_{ni}\,x_{mi} & x^2_{mi} & \\ -y_{mi}\,x_{nj} & y_{mi}\,y_{ni} & y_{mi}\,x_{mi} & -y^2_{mi} & y_{mi}\,x_{nj} & -y_{mi}\,y_{ni} & -y_{mi}\,x_{mi} & y^2_{mi} \end{bmatrix}$$

where $x_{nj} = x_n - x_j$ etc.

$$A = (x_{ji}\,y_{mi} + x_{mi}\,y_{nj} - x_{ni}\,y_{mi})$$

$$t = \text{panel thickness}$$

In its local co-ordinates the panel is arranged as in Figure 8

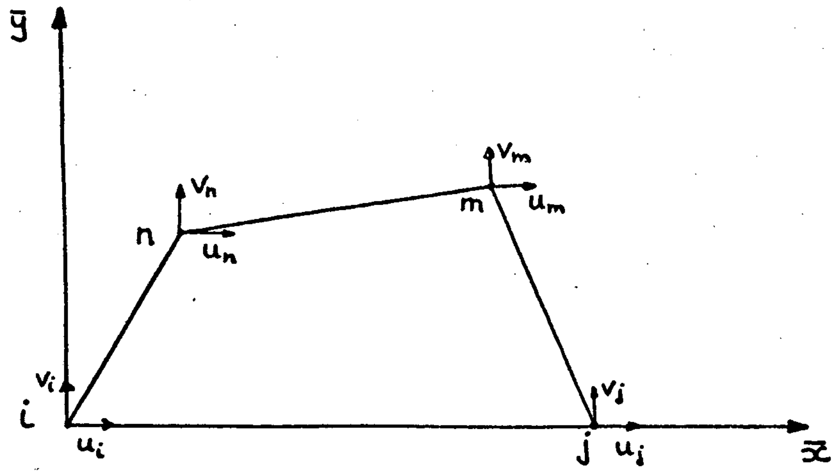


Figure 8

It must be pointed out here that the shear panel and boom idealization of an aircraft structure is a rough approximation of the real thing, and the plane stress elements should be used rather than the shear panels. A comparison of the results obtained from both configurations will be given in the Experimental Section B of this thesis.

A2.4 The Transformation Matrices

A2.4.1 Transformation Matrix for the End-load element

Consider Figure 9 where the rod-element is in three dimensional space with the set of axes at the "i" end of the rod parallel to the global axes. Only two angles of rotation define the elements orientation relative to the global axes.

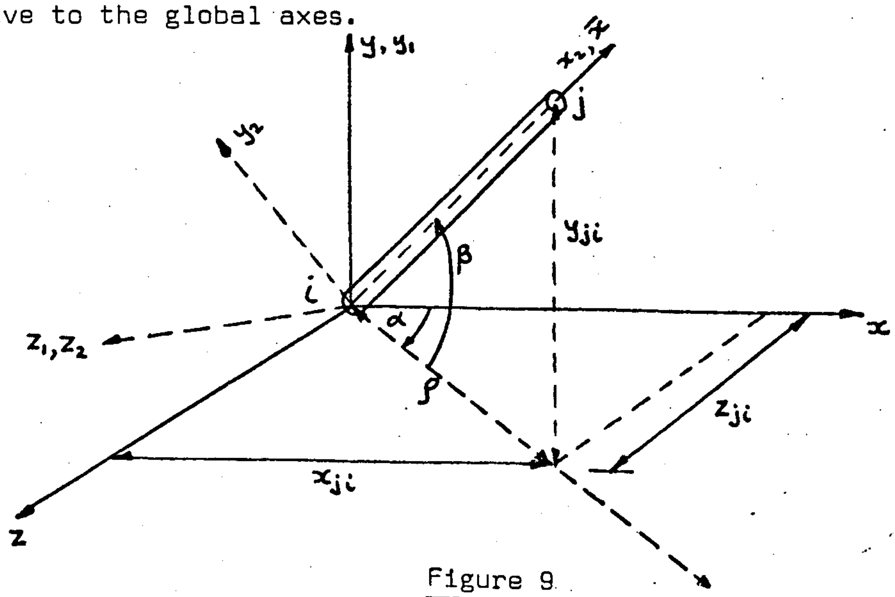


Figure 9

If the element is first rotated about the y-axis through an angle α then the relationship between the new co-ordinates x_1, y_1, z_1 and x, y, z is

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 & \sin\alpha \\ 0 & 1 & 0 \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{a2.4.1}$$

further rotation about the z_1 axis through an angle β yields the relationship between x_2 y_2 z_2 and x_1 y_1 and z_1.

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{a2.4.2}$$

Substituting a2.4.1 into a2.4.2 yields

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \cos\beta\sin\alpha \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

or in matrix notation

$$(\bar{\Gamma}) = (T)(\Gamma) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{a2.4.3}$$

where (T) is called the transformation matrix. Now if we let barred quantities be local deflections and forces then

$$(\bar{u}) = (T)(u) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \text{a2.4.4}$$

and $(\bar{p}) = (\bar{k})(\bar{u})$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ a2.4.5

also $(\bar{p}) = (T)(p)$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ a2.4.6

Then combining these relationships yields

$$(T)(p) = (\bar{k})(T)(u)$$

or

$$(p) = (T)^{-1}(\bar{k})(T)(u) \qquad \text{.................... a2.4.7}$$

The expression $(T)^{-1}(\bar{k})(T)$ is called the global stiffness matrix and (p) and (u) are in global axis directions. Thus we have once again the familiar equation

$$(p) = (k)(u)$$

The fortunate situation exists, in that, the transformation matrix does not have to be inverted but only transposed because it is orthogonal i.e.

$$|T| = 1$$

thus

$$(k) = (T)^T(k)(T) \qquad \text{.................... a2.4.8}$$

Now considering Figure 9 once more, expressions for the trigonometric functions of the angles can be found from the co-ordinates of the two nodal points.

Let $z_{ji} = z_j - z_i$ etc.

$$\rho = (z^2_{ji} + x^2_{ji})^{\frac{1}{2}}$$

and

$$L = (x^2_{ji} + y^2_{ji} + z^2_{ji})^{\frac{1}{2}}$$

then $\sin\alpha = \dfrac{z_{ji}}{\rho}$; $\cos\alpha = \dfrac{x_{ji}}{\rho}$

$\sin\beta = \dfrac{y_{ji}}{L}$; $\cos\beta = \dfrac{\rho}{L}$

and the transformation matrix becomes

$$(T) = \begin{bmatrix} \dfrac{x_{ji}}{L} & \dfrac{y_{ji}}{L} & \dfrac{z_{ji}}{L} \\ -\dfrac{x_{ji}\,y_{ji}}{L} & \dfrac{\rho}{L} & \dfrac{y_{ji}\,z_{ji}}{L\rho} \\ -\dfrac{z_{ji}}{\rho} & 0 & \dfrac{x_{ji}}{\rho} \end{bmatrix}$$

Now in order that the matrix multiplication of equation a2.4.8 may take place the stiffness matrix of the end load element must be made three dimensional. The endload element is one dimensional i.e. it only has stiffness in the $\bar{x}$ direction thus the stiffnesses in the $\bar{y}$ and $\bar{z}$ directions are zero, therefore equation a2.3.6 becomes

$$[\bar{k}] = \frac{EA}{L}\begin{bmatrix} 1 & & & & & \\ 0 & 0 & & & \text{sym.} & \\ 0 & 0 & 0 & & & \\ -1 & 0 & 0 & 1 & & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and in this form the multiplication can take place.

$$[k] = \begin{bmatrix} [T] & 0 \\ 0 & [T] \end{bmatrix}^T [\bar{k}] \begin{bmatrix} [T] & 0 \\ 0 & [T] \end{bmatrix}$$

A2.4.2 <u>Transformation Matrix for a Plane Element</u>

Any three points on a plane surface are sufficient to define its orientation in 3-D space, thus only 3 corners of a quadrilateral need be used to define its co-ordinate transformation matrix.

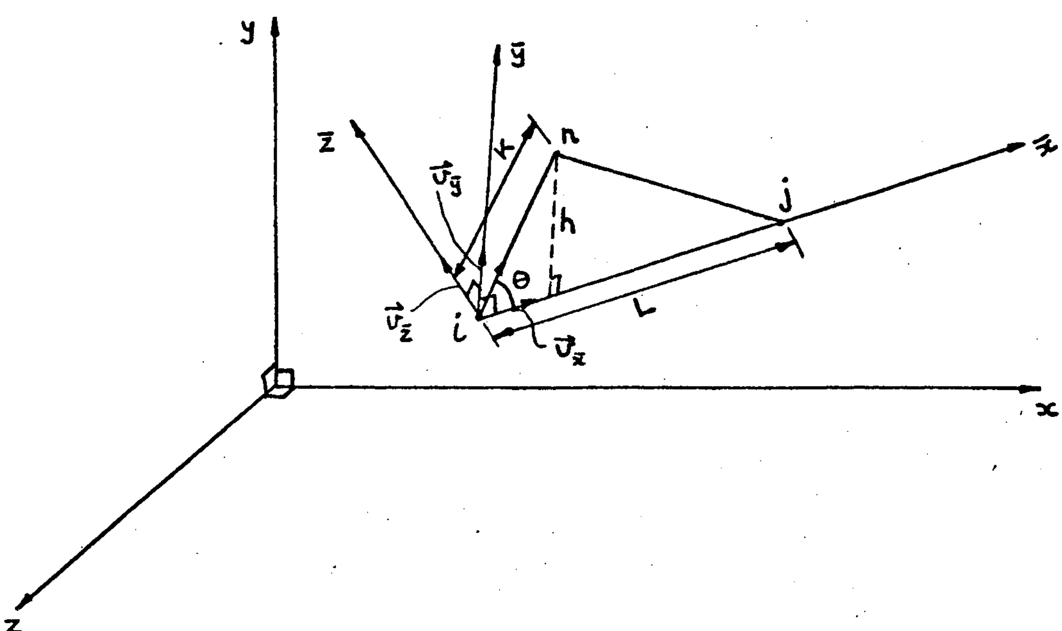


Figure 10

Consider in Figure 10 a triangular plane i j n with its local $\bar{x}$ axis lying along the i j side and the $\bar{y}$ and $\bar{z}$ axes at right angles to it as shown. By obtaining the direction cosines of the unit vectors $\bar{v}_{\bar{x}}$, $\bar{v}_{\bar{z}}$ and $\bar{v}_{\bar{y}}$ the transformation of the co-ordinates is complete.

Let $\qquad y_{ji} = y_j - y_i$

$\qquad z_{ni} = z_n - z_i \quad$ etc.

then the vector representing side i j is

$$\vec{V}_{\bar{x}} = i\, x_{ji} + j\, y_{ji} + k\, z_{ji}$$

from vector theory and its unit vector

$$\bar{v}_{\bar{x}} = i\,\frac{x_{ji}}{L} + j\,\frac{y_{ji}}{L} + k\,\frac{z_{ji}}{L}$$

where $\qquad L = (x^2_{ji} + y^2_{ji} + z^2_{ji})^{\frac{1}{2}}$

The vector representing side i n is

$$\vec{V}_{in} = i\, x_{ni} + j\, y_{ni} + k\, z_{ni}$$

and its unit vector

$$\vec{v}_{in} = i\,\frac{x_{ni}}{K} + j\,\frac{y_{ni}}{K} + k\,\frac{z_{ni}}{K}$$

where $\qquad K = (x^2_{ni} + y^2_{ni} + z^2_{ni})^{\frac{1}{2}}$

Now the cross product of two vectors which have an angle θ between them is given by

$$\vec{A} \times \vec{B} = |A|.|B| \sin\theta.\vec{u}$$

where $\vec{u}$ is a unit vector perpendicular to the plane made by vectors $\vec{A}$ and $\vec{B}$.

Now cross multiplication of $\vec{v}_{\bar{x}} \times \vec{v}_{in}$ and deviding by $\sin\theta$ yields $\vec{v}_{\bar{z}}$

$$\vec{v}_{\bar{z}} = \frac{1}{\sin\theta}\begin{bmatrix} i & j & k \\ \frac{x_{ji}}{L} & \frac{y_{ji}}{L} & \frac{z_{ji}}{L} \\ \frac{x_{ni}}{K} & \frac{y_{ni}}{K} & \frac{z_{ni}}{K} \end{bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad a2.4.9$$

From Figure 10 it can be seen that

$$\sin\theta = \frac{h}{K}$$

thus multiplying out equation a2.4.9, yields

$$\vec{v}_{\bar{z}} = i\,\frac{y_{ji}\,z_{ni} - y_{ni}\,z_{ji}}{L\,h} + j\,\frac{z_{ji}\,x_{ni} - x_{ji}\,z_{ni}}{L\,h} + k\,\frac{x_{ji}\,y_{ni} - y_{ji}\,x_{ni}}{L\,h}$$

and similarly

$$\vec{v}_{\bar{y}} = \vec{v}_{\bar{z}} \times \vec{v}_{\bar{x}}$$

Now we can write

$$\vec{v}_{\bar{x}} = i\,\lambda_{\bar{x}x} + j\,\lambda_{\bar{x}y} + k\,\lambda_{\bar{x}z}$$

$$\vec{v}_{\bar{y}} = i\,\lambda_{\bar{y}x} + j\,\lambda_{\bar{y}y} + k\,\lambda_{\bar{y}z}$$

$$\vec{v}_{z} = i\,\lambda_{\bar{z}x} + j\,\lambda_{\bar{z}y} + k\,\lambda_{\bar{z}z}$$

thus the transformation matrix for a plane element is

$$(T) = \begin{bmatrix} \lambda_{\bar{x}x} & \lambda_{\bar{x}y} & \lambda_{\bar{x}z} \\ \lambda_{\bar{y}x} & \lambda_{\bar{y}y} & \lambda_{\bar{y}z} \\ \lambda_{\bar{z}x} & \lambda_{\bar{z}y} & \lambda_{\bar{z}z} \end{bmatrix}$$

The plane element only has stiffness in its own plane so the local co-ordinate matrix must also be "prepared" by adding rows and colums of zeros in the $\bar{z}$ direction before multiplication with the (T) matrix takes place.

A3. THE AIRSTR COMPUTER PROGRAM

A3.1 Why write another one?

There is no better way of mastering a subject other than applying the theory oneself. The program was started, and built up on, as the lectured course in finite elements progressed. Many problems were encountered along the way and solved in one way or another.

The program is by no means a highly efficient piece of programming, however, for the smaller type of exercise used say by undergraduate students in aircraft structures it is faster than using a large system like ASKA and requires less pre-use reading of manuals etc.

A3.2 Description of Method

The entire program is executed in core i.e. no data is read to or from files, thus eliminating almost entirely Input/Output time which is very costly.

The program has a main driver routine which calls all the subroutines necessary, in order. A main driver card tells the program how many nodal points are used and how many of each element is used. The nodal point co-ordinates are read in first and stored in column arrays. Each element is read in on a single card and its stiffness matrix, in local co-ordinates, assembled, prepared (zeros added), transformed and then packed into the (K) matrix in banded form (Ref. 3). The known deflections and nodal point loads are added and the set of equations solved. The solved deflections are correctly assosiated with the nodal points at which they occur and stored. Each element card is then re-read and the deflections used to calculate the stresses and strains in each element.

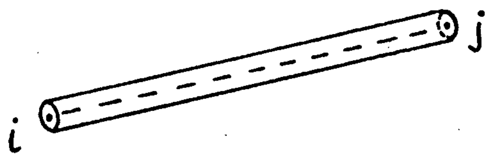
There is a full "data echo" of all the input data. The output consists of a title heading for the job, the nodal point deflections and the stresses in each element, all given in clearly legible format.

A complete listing of the program can be found in Appendix A1.

A3.3 Elements Available in AIRSTR

A3.3.1 End-load or Rod element

Can be arbitrarily orientated in space.



Input Nodal point numbers i and j
Crosssectional area A_f
E - Youngs Modulus.

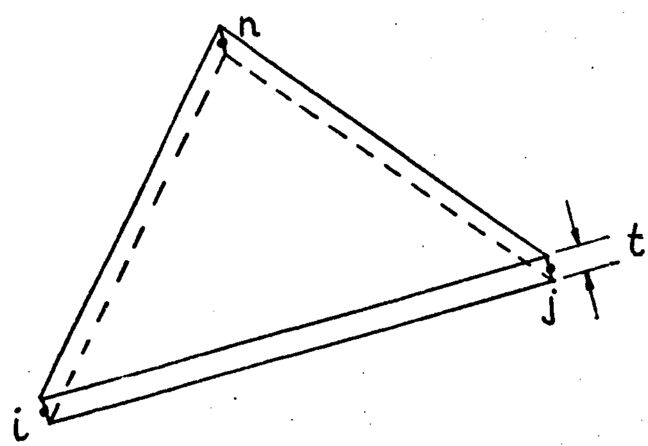
Output $\sigma_{\overline{xx}}$ Axial stress
$F_{\overline{x}}$ Axial force

For a linearly tapered element the mean cross-sectional area may be used

$$A_f = \frac{A_{fi} + A_{fj}}{2}$$

A3.3.2 C.S.T. Element (Constant Strain Triangle)

This element can be arbitrarily orientated in space, and can either be used for plane stress or plane strain analysis.



Input Nodal point numbers i, j and n

t - panel thickness

ν - Poissons ratio

E - Youngs Modulus.

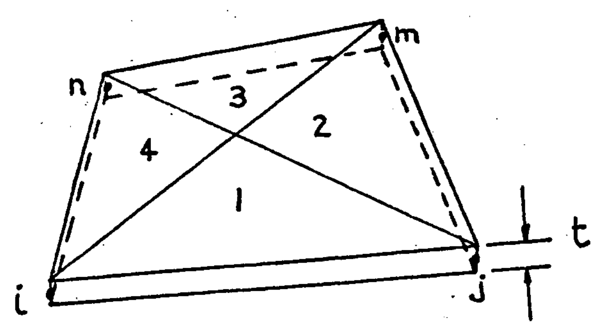
Output $\varepsilon_{\bar{x}\bar{x}}$, $\varepsilon_{\bar{y}\bar{y}}$, $\varepsilon_{\bar{x}\bar{y}}$, $\sigma_{\bar{x}\bar{x}}$, $\sigma_{\bar{y}\bar{y}}$, $\sigma_{\bar{x}\bar{y}}$ with i - j as $\bar{x}$-axis

If a variable thickness panel is used a mean value for the thickness may be chosen.

$$t = \frac{t_i + t_j + t_n}{3}$$

A3.3.3 4- C.S.T. Quadrilateral Element

A four sided plane stress membrane element arbitrarily orientated in space.



It is made up of 4-C.S.T. elements with the dummy point lying at the intersection of the diagonals. The centre point is eliminated by putting the forces there equal to zero.

Input Nodal point numbers i, j, m and n

t - panel thickness

ν - Poissons ratio

E - Youngs Modulus.

Output $\varepsilon_{\bar{x}\bar{x}}$, $\varepsilon_{\bar{y}\bar{y}}$, $\varepsilon_{\bar{x}\bar{y}}$, $\sigma_{\bar{x}\bar{x}}$, $\sigma_{\bar{y}\bar{y}}$, $\sigma_{\bar{x}\bar{y}}$

for each triangle with the $\bar{x}$-axis parallel to side i-j. The mean stresses of the 4 triangles are also given.

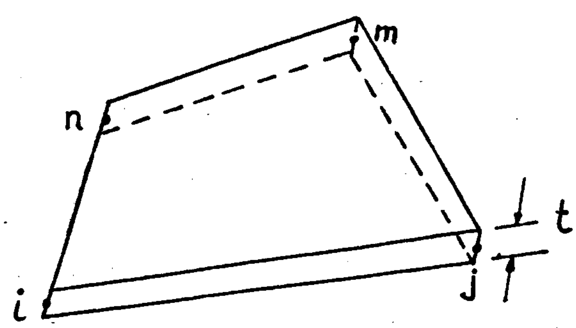
A3.3.4 Quadrilateral Plane Stress Element

Basically the same as the previous element. Only stress output at the centre of the element

$$\sigma_{\bar{x}\bar{x}} \quad \sigma_{\bar{y}\bar{y}} \quad \sigma_{\bar{x}\bar{y}}$$

A3.3.5 Shear Quadrilateral Panel Element

The panel is assumed to carry shear stresses only which are constant throughout the panel. Arbitrary orientation in space.



Input Nodal point numbers i, j, m and n

t - panel thickness

ν - Poissons Ratio

E - Youngs Modulus

Output $\varepsilon_{\bar{x}\bar{y}}$ and $\sigma_{\bar{x}\bar{y}}$ constant throughout panel.

Depending on the variation in thickness in a quadrilateral panel dis= cretion must be used in choosing a representative thickness (t).

A3.3.6 Idealized Panel Element

The program automatically generates an idealized panel as outlined in Ref. 5, in which, the following approximation is made:

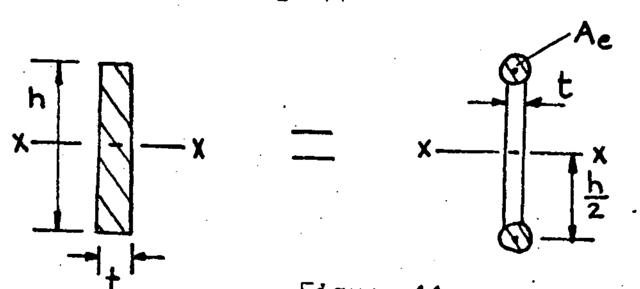


Figure 11

Moment of inertia about the x-x axis for both cross-sections must be equal

$$\therefore \qquad \frac{1}{12} t h^3 = 2.A_e \left(\frac{h}{2}\right)^2$$

$$\therefore \qquad A_e = \frac{1}{6} t h$$

Now considering the quadrilateral element in Figure 12

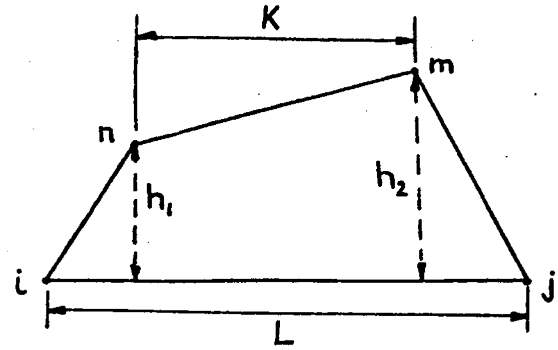


Figure 12

The idealized panel is made up of a shear panel as described in A3.3.5 plus 4 end load elements connecting the corner nodal points i-j, n-m, i-n, j-m. The equivalent areas (A_e) of rods i-j and n-m are equal, and i-n and j-m are equal.

For ij and nm

$$A_e = \frac{\frac{1}{6} t h_1 + \frac{1}{6} t h_2}{2}$$

$$= \frac{t (h_1 + h_2)}{12}$$

and i-n and j-m

$$A_e = \frac{\frac{1}{6} t L + \frac{1}{6} t K}{2}$$

$$= \frac{t(L + K)}{12}$$

This idealization satisfies the special case where the panel is a rectangle.

Input Nodal point numbers i, j, m, n

t - panel thickness

ν - Poissons ratio

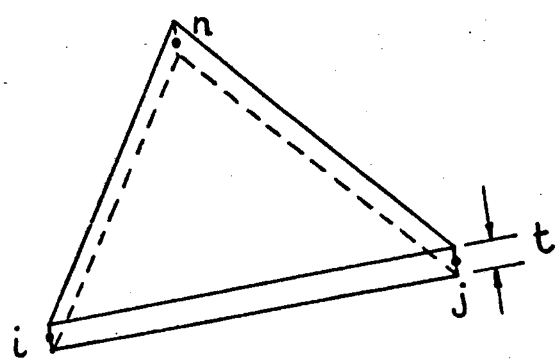
E - Youngs Modulus.

Output Same as for Shear Panel and End-load elements.

The option of having Shear Panels and End-load elements separately available gives the program flexibility in that the programmer may wish to make his own idealization of a structure.

A3.3.7 Shear Triangle Panel Element

This is an approximate element as described in section A2.3.4 and only carries shear stresses. It must be accompanied by rod elements to give it, in-plane direct stress stiffness.



Input Nodal point Numbers i, j and n

t - panel thickness

ν - Poissons Ratio

E - Youngs Modulus

Output Same as for C.S.T. element. Ignore direct stresses and strains.

The program will reject badly shaped triangular elements (C.S.T. and Shear triangles), i.e. if the perpendicular from nodal point n to line i-j does not lie between the points i and j. The programmer should remember this when drawing up his finite element grid.

A3.3.8 Idealized Triangular Panel Element

There is no suggested method for idealizing a triangular panel element so an attempt has been made by the writer.

Consider the triangular panel shown in Figure 13.

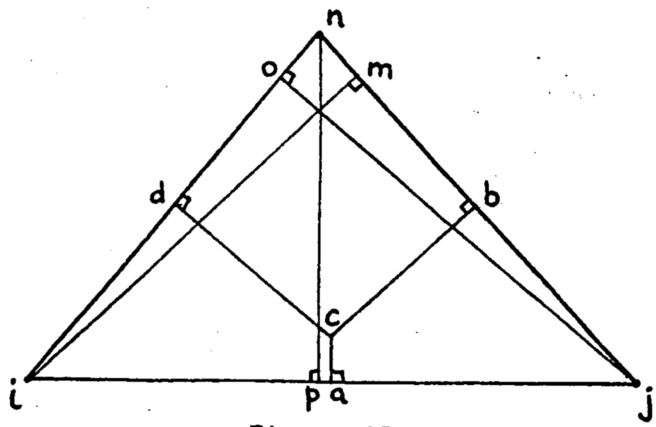


Figure 13

Let $nb = bj$

$nd = di$

and $ia = aj$

The moment of inertia of the triangle is taken about the point C and a mean height h' taken.

$$h' = (oj + mi + pn)/3$$

Then making the rod elements i-n, n-j and i-j all of the same equivalent cross-sectional area yields

$$\frac{1}{12} t (h')^3 = A_e\left[(ac)^2 + (bc)^2 + (dc)^2\right]$$

$$\therefore \quad A_e = \frac{t (h')^3}{12(ac^2 + bc^2 + dc^2)}$$

Thus the idealized triangular panel element consists of a triangular shear panel stiffened by rod elements of area A_e between the three nodal points.

Input Same as for Shear Triangle Element.

Output Same as for Shear Triangle and three rod elements.

A3.4 Use of the AIRSTR program

One run of the program will do a single static load case on a semi-mono= coque type structure. The dimensioned stiffness matrix limits the number of nodal points to 100 and a bandwidth of 190 i.e. in the numbering of the nodal points no two connected nodal points may differ in value by more than 63. The program was written and developed on a UNIVAC 1110 computer using the EXEC 8 system. All the data cards are written in free format i.e. the values are separated by commas on each card. Floating point values must have a decimal point and integers not.

The data deck of cards is given below. The element cards must be arranged in the order specified. To comply with their stiffness derivation, ele= ments must be numbered in an anticlockwise direction as indicated in sec= tion A3.3

The deck is headed by the identification of the user and his account number, called the @RUN card.

@RUN ...

The elements are read twice, so they are stored in a temporary file called in EXEC 8 an ELEMENT; So the element cards are headed by say

@ELT,I..ELMNTS

Then follows the data cards for the elements. Each element type is grouped together and regardless of the types of elements used the order of sequence in which they appear in the data deck must be as follows. One card for each element

i, j, A_f, E-modulus	(End-load or Rod Elements).
i, j, n, t, ν, E	(Plane Stress C.S.T. elements)
i, j, n, t, ν, E	(Plane Strain C.S.T. elements)
i, j, m, n, t, ν, E	(4-C.S.T. Plane Stress Quadrilaterals)
i, j, m, n, t, ν, E	(Plane Stress Quad. element)
i, j, m, n, t, ν, E	(Shear Quad. Panel elements)
i, j, m, n, t, ν, E	(Idealized Quad. Panel elements)
i, j, n, t, ν, E	(Shear Triangle Panel elements)
i, j, n, t, ν, E	(Idealized Triangle Panel elements)

That is the end of the elements in the structure. Next comes the execution card which starts the run of the program which is stored on file.

@XQT␣␣EDDY.AIRSTR

The first card the program reads is the title of the job submitted. The complete card, i.e. 80 spaces, is available for the title, e.g.

STARBOARD␣WING␣LANDING-LOAD␣CASE-1

The Main Driver card follows next. This card tells the program how many nodal point co-ordinates, and elements of each type to read. All the numbers on this card are integer values. Once again the order of the numbers must be as shown, and if an element of a certain type is not be used, a zero must be inserted in the correct position.

N,NRØD3,NCST,NTRIM,NQAT,NQUA4,NSHTR,NIPAN,NTRIS,NIDTS

Where	N	- No. of Nodal points
	NROD3	- " " End-load elements
	NCST	- " " Plane Stress C.S.T. elements
	NTRIM	- " " Plane Strain C.S.T. elements
	NQAT	- " " 4-C.S.T. Plane Stress Quadrilaterals
	NQUA4	- " " Plane Stress Quadrilateral Elements
	NSHTR	- " " Shear Quadrilateral Panel Elements
	NIPAN	- " " Idealized Quadrilateral Panel Elements
	NTRIS	- " " Shear Triangle Panel Elements
	NIDTS	- " " Idealized Triangle Panel Elements.

The nodal point co-ordinates are read next.

I ,	XX(I) ,	YY(I) ,	ZZ(I)
Nod. pt. number,	x-coord,	y-coord,	z-coord.

The nodal point number is an integer and the co-ordinates floating point. They do not have to be read in in order i.e. 1 to N. If a two dimensional structure is being analysed let all the z-co-ordinates be zero (0.0)

The structure stiffness matrix is now assembled so the element data must be read in, this is simply done with the following card.

@ADD␣␣ELMNTS

A secondary driver card is inserted next to tell the program how many nodal point loads and displacements it must read. The card description is self explanatory.

ILØAD, IDISP

It now follows that the loads are defined. Each nodal point load is given on a separate card as follows.

Nodal Point No., Direction, Magnitude
INØD , ID , ALØAD

The load magnitudes are positive in the direction of the chosen global axes. Should the load at a particular nodal point not lie parallel with any of the global axes, then that load must be split up into its components parallel with global axes and each component read in on a separate card. The direction identification is as follows

ID = 1 - x-axis (global)
ID = 2 - y-axis (global)
ID = 3 - z-axis (global)

For a structure to be in static equilibrium either all the forces and moments on it must balance or it must have supports to ballance the applied loads. The supports can either be assumed to be rigid i.e. they deflect very little in comparison with the rest of the structure or they can deflect a known amount. These then are the known displace= ments, which are, like the loads, each read in on separate cards.

Nodal Point No, Direction, Magnitude
INØD , ID , DISP

If a two dimensional structure is being analysed (x-y plane) then all the nodal point deflections in the z-direction must be set to zero.

The program now solves the set of equations, and to complete the back substitution to determine the stresses in each element, they are read again by insertirg the card

@ADD..ELMNTS

and the run is terminated by the

@FIN card.

An example structure with its data deck listing and print out can be found in Appendix A2.

Footnote:

The load vector is zeroed in the program before the applied loads are read, so nodal points which do not have loads applied to them, do not need zero load cards.

If at a particular point in a structure the deflection is known, but the applied loads unknown, then the no of loads (ILOAD) may be made zero and the known deflection inserted. The stresses in the structure will be output for such a condition.

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6. S.P. Timoshenko and J.N. Goodier "Theory of Elasticity" Third Edition, Mc Graw-Hill Book Company.

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APPENDIX A1

Listing of the AIRSTR computer program

```
C**  PROGRAMMER E.M.E.BAUMGARTNER.          DRIVER  PROGRAM
C**  ENGINEERING MASTERS THESIS.
C**STATIC FINITE ELEMENT ANALYSIS OF SEMI-MONOCOQUE AIRCRAFT STRUCTURES
C***************************************************************
      COMPILER(XM=1)
      DIMENSION A(300,190),B(300)
      DIMENSION U(3,100),XX(100),YY(100),ZZ(100)
      COMMON A
      NODAL = 100
      NBAND = 190
      IMATK = NODAL * 3
C**  READ IN TITLE CARD
      READ(5,5)
      WRITE(6,15)
      WRITE(6,5)
C**  READ MAIN DRIVER CARD
      READ(5,10)N,NROD3,NCST,NTRIM,NQAT,NQUA4,NSHTR,NIPAN,NTRIS,NIDTS
      WRITE(6,11)N,NROD3,NCST,NTRIM,NQAT,NQUA4,NSHTR,NIPAN,NTRIS,NIDTS
      NN=3*N
C*
C*
C**  READ IN NODAL POINT COORDINATES
C*
      READ(5,10)(J,XX(J),YY(J),ZZ(J),I=1,N)
      WRITE(6,13)
      WRITE(6,12)(J,XX(J),YY(J),ZZ(J),J=1,N)
C**  VACATE STIFFNESS AND LOAD MATRIX
C*
      DO 100 I=1,NN
      DO 100 J=1,NBAND
      A(I,J) = 0
      B(I) = 0
  100 CONTINUE
C**  ASSEMBLY OF STRUCTURE STIFFNESS MATRIX
  301 IF(NROD3.EQ.0) GO TO 305
      CALL RODPAK(NROD3,IMATK,NODAL,A,XX,YY,ZZ,NBAND)
  305 IF(NCST.EQ.0) GO TO 308
      JSTRS = 1
      CALL TRIM3 (NCST,IMATK,NODAL,A,XX,YY,ZZ,NBAND,JSTRS)
  308 IF(NTRIM.EQ.0) GO TO 309
      JSTRN = 0
      CALL  TRIM3(NTRIM,IMATK,NODAL,A,XX,YY,ZZ,NBAND,JSTRN)
  309 IF(NQAT.EQ.0) GO TO 310
      CALL  QUAPAC(NQAT,IMATK,NODAL,A,XX,YY,ZZ,NBAND)
  310 IF(NQUA4.EQ.0) GO TO 311
      CALL  QUA4(NQUA4,IMATK,NODAL,A,XX,YY,ZZ,NBAND)
  311 IF(NSHTR.EQ.0) GO TO 314
      ISH = 0
      CALL SHTRAP(NSHTR,IMATK,NODAL,A,XX,YY,ZZ,ISH,NBAND)
  314 IF(NIPAN.EQ.0) GO TO 315
      IID = 1
      CALL SHTRAP(NIPAN,IMATK,NODAL,A,XX,YY,ZZ,IID,NBAND)
  315 IF(NTRIS.EQ.0) GO TO 318
      ITS = 0
      CALL  TRISH(NTRIS,IMATK,NODAL,A,XX,YY,ZZ,ITS,NBAND)
  318 IF(NIDTS.EQ.0) GO TO 330
      ITD = 1
      CALL TRISH(NIDTS,IMATK,NODAL,A,XX,YY,ZZ,ITD,NBAND)
C*
C**  SECONDARY DRIVER CARD. NO. OF LOADS AND DISPLACEMENTS
C*
```

```
  330 READ(5,40) ILOAD,IDISP
C**  LOADS
C*
      IF(ILOAD.EQ.0) GO TO 705
      WRITE(6,500)
      DO 700 ILD=1,ILOAD
      READ(5,50) INOD,ID,ALOAD
      WRITE(6,505)INOD,ID,ALOAD
      IF(INOD.LE.N.AND.ID.LE.3) GO TO 690
      WRITE(6,680)
      GO TO 1000
  690 IACOL=(INOD-1)*3+ID
      B(IACOL) = ALOAD
  700 CONTINUE
C*
C**  DEFLECTIONS OR BOUNDARY CONDITIONS
C*
  705 WRITE(6,510)
      CALL  SUPORT(A,B,IMATK,NBAND,IDISP)
C**  SOLVE SET OF EQUATIONS
      CALL  SOLVE(A,B,NN,IMATK,NBAND)
      WRITE(6,60)
      WRITE(6,5)
      NFN = NN-IDISP
C*
C**  SET THE DEFLECTIONS TO THE CORRECT NODAL POINTS
C*
  920 DO 925 JNOD=1,N
      DO 925 JDIR=1,3
      ND= (JNOD-1)*3+JDIR
      U(JDIR,JNOD) = B(ND)
  925 CONTINUE
      WRITE(6,520)
      WRITE(6,521)
      WRITE(6,525)(J,(U(I,J),I = 1,3),J = 1,N)
C*
C**  CALCULATION OF THE LOADS IN THE ELEMENTS.
C**  *****************************************
C**  RE-READ THE DATA FOR EACH ELEMENT AND DETERMINE THE STRESSES
C*
      WRITE(6,60)
      WRITE(6,5)
      WRITE(6,28)
C**  ENDLOAD ELEMENTS
  951 IF(NROD3.EQ.0) GO TO 960
      WRITE(6,21)
      CALL RODES3(NROD3,U,NODAL,XX,YY,ZZ)
C**  CALCULATION OF STRESSES IN C.S.T. ELEMENTS (PLANE STRESS)
  960 IF(NCST.EQ.0) GO TO 966
      IPS = 1
      CALL CONTES(NCST,U,NODAL,XX,YY,ZZ,IPS)
C**  CALCULATION OF STRESSES IN C.S.T.ELEMENTS (PLANE STRAIN)
  966 IF(NTRIM.EQ.0) GO TO 968
      IPST = 0
      CALL CONTES(NTRIM,U,NODAL,XX,YY,ZZ,IPST)
C**  4-CST QUADRILATERALS (PLANE STRESS)
  968 IF(NQAT.EQ.0) GO TO 970
      CALL QUASTR(NQAT,U,NODAL,XX,YY,ZZ)
C**  CALCULATION OF STRESSES IN QUA4 ELEMENT
  970 IF(NQUA4.EQ.0) GO TO 975
```

```
      CALL  Q4ST(NQUA4,U,NODAL,XX,YY,ZZ)
C**   CALCULATION OF STRESSES IN SHEAR QUADRILATERAL ELEMENTS
  975 IF(NSHTR.EQ.0) GO TO 980
      ISH = 0
      WRITE(6,60)
      WRITE(6,22)
      CALL PANSTR(NSHTR,U,NODAL,XX,YY,ZZ,ISH)
C**   IDEALIZED QUADRILATERAL PANELS
  980 IF(NIPAN.EQ.0) GO TO 985
      WRITE(6,60)
      WRITE(6,22)
      IDP = 1
      CALL PANSTR(NIPAN,U,NODAL,XX,YY,ZZ,IDP)
C**   SHEAR TRIANGLES
  985 IF(NTRIS.EQ.0) GO TO 990
      KID = 0
      CALL  CONTES(NTRIS,U,NODAL,XX,YY,ZZ,KID)
      WRITE(6,965)
C**   IDEALIZED TRIANGLES
  990 IF(NIDTS.EQ.0) GO TO 1000
      KIDTS = 2
      CALL  CONTES(NIDTS,U,NODAL,XX,YY,ZZ,KIDTS)
      WRITE(6,965)
    5 FORMAT(80H
     C                                              )
   10 FORMAT(   )
   11 FORMAT(10X,10I5)
   12 FORMAT(10X,I6,3E13.4)
   13 FORMAT(10X,'NODAL POINT COORDINATES',/)
   15 FORMAT(///,24X,'*** DATA ECHO ***'//)
   21 FORMAT(2X,'ROD NOD.PTS',3X,'   AREA    ',4X,'  E-VALUE ',4X,'   LEN
     CGTH  ',4X,'AXIAL LOAD',4X,'  STRESS   ',/)
   22 FORMAT(2X,'SHEAR PANEL NOD.PTS',4X,' B-LENGTH ',4X,'  HEIGHT  ',3X
     C,'  G-VALUE ',2X,' THICKNESS',4X,'  EPSXY    ',2X,'SHEAR STRESS',/)
   28 FORMAT(46X,'SEMI-MONOCOQUE STRUCTURAL  ANALYSIS',/,46X,36('*'))
   40 FORMAT( )
   50 FORMAT( )
   60 FORMAT('1')
  500 FORMAT(10X,'LOAD DATA,--NOD. PT.,DIRECTION,MAGNITUDE',/)
  505 FORMAT(10X,2I6,E10.4)
  510 FORMAT(10X,'KNOWN DEFLECTIONS',/)
  520 FORMAT(30X,'NODAL POINT DEFLECTIONS',/,28X,25('*'),/)
  521 FORMAT(/,9X,'NODAL PT.',8X,'X-DEFL',7X,'Y-DEFL',7X,'Z-DEFL',/)
  525 FORMAT(10X,I6,4X,3E13.4,/)
  680 FORMAT(30X,'*** ERROR- CHECK YOUR LOAD INPUT DATA',//)
  965 FORMAT(10X,'THESE PANELS ONLY CARRY SHEAR.IGNORE OTHER OUTPUT.')
 1000 CALL EXIT
      END

      SUBROUTINE CONTES(NCST,U,NODAL,XX,YY,ZZ,JID)
C**   DETERMINATION OF STRESS AND STRAIN IN C.S.T. ELEMENTS,SHEAR
C**   TRIANGLES AND IDEALIZED TRIANGLES
      DIMENSION NOD(3),U(3,NODAL),V(12,12),VT(12,12),T(12,12)
      DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL)
      WRITE(6,16)
      WRITE(6,17)
      DO 50 K = 1,NCST
      READ(5,25)IC,JC,NC,TH,ANU,EE
      NOD(1) = IC
      NOD(2) = JC
      NOD(3) = NC
C**   SET UP LOCAL COORDINATE SYSTEM
```

```
      XJI = XX(JC) - XX(IC)
      XNI = XX(NC) - XX(IC)
      XNJ = XX(NC) - XX(JC)
      YJI = YY(JC) - YY(IC)
      YNI = YY(NC) - YY(IC)
      YNJ = YY(NC) - YY(JC)
      ZJI = ZZ(JC) - ZZ(IC)
      ZNI = ZZ(NC) - ZZ(IC)
      ZNJ = ZZ(NC) - ZZ(JC)
      EL = SQRT(XJI**2 + YJI**2 + ZJI**2)
      EK = SQRT(XNI**2 + YNI**2 + ZNI**2)
      EH = SQRT(XNJ**2 + YNJ**2 + ZNJ**2)
      WHY = (EK**2 + EL**2-EH**2)/(2.0*EL)
      H1 = SQRT(EK**2 - WHY**2)
      AREA = 0.5*EL*H1
C**   LOCAL COORDINATE DIMENSIONS
      XBI = 0
      YBI = 0
      XBJ = EL
      YBJ = 0
      XBN = WHY
      YBN = H1
      XBJI = XBJ - XBI
      XBNJ = XBN  - XBJ
      XBNI = XBN - XBI
      YBNI = YBN - YBI
       DO 10 JC = 1,3
      NK = NOD(JC)
      DO 10 K2 = 1,3
      V(K2,JC) = U(K2,NK)
   10 CONTINUE
      CALL PLATRA(IC,JC,NC,XX,YY,ZZ,NODAL,T)
      CALL AMTMUL(3,3,T,V,VT,12)
      EPSXX = (VT(1,2) - VT(1,1))/XBJI
      EPSYY = (VT(2,3)*XBJI + VT(2,1)*XBNJ - VT(2,2)*XBNI)/(XBJI*YBNI)
      EPSXY=(VT(1,3)*XBJI+VT(1,1)*XBNJ-VT(1,2)*XBNI)/(XBJI*YBNI)
     C+(VT(2,2) - VT(2,1))/XBJI
      IF(JID.EQ.1) GO TO 40
C**  PLANE STRAIN ANALYSIS
      A = (1.0 - ANU)*EE/((1.0 + ANU)*(1.0 - 2.0*ANU))
      SIGX = A*(EPSXX + ANU/(1.0 - ANU)*EPSYY)
      SIGY = A*(EPSYY + ANU/(1.0 - ANU)*EPSXX)
      GO TO 45
C**  PLANE STRESS ANALYSIS
   40 A = EE/(1.0-ANU**2)
      SIGX = A*(EPSXX + ANU*EPSYY)
      SIGY = A*(EPSYY + ANU*EPSXX)
   45 SHEAR = EE/(2.0*(1.0 + ANU))*EPSXY
      WRITE(6,15)(NOD(I),I=1,3),EPSXX,EPSYY,EPSXY,SIGX,SIGY,SHEAR
      IF(JID.LT.2) GO TO 50
C** IDEALIZED TRIANGLE STRESS RETRIEVAL
      WRITE(6,27)
      WRITE(6,21)
      COST = WHY/EK
      COTT = WHY/H1
      TANT = 1.0/COTT
      SINT = H1/EK
      TANPH = H1/(EL-WHY)
      COSPH = (EL-WHY)/EH
      SINPH = H1/EH
```

```
      AC = 0.5*(EK/COST - EL)*COTT
      DC = 0.5*(EK*TANT - (EK/COST - EL)/SINT)
      BC = 0.5*(EH*TANPH - (EH/COSPH - EL)/SINPH)
      HDASH = 2./3.*AREA*(1./EL + 1./EK + 1./EH)
      RA = TH*HDASH**3/(12.0*(AC**2+DC**2+BC**2))
C**   IDEALIZED TRIANGLE STIFFENER LOADS
C**   ROD NO. 1
      Q2 = (U(1,JC)*XJI + U(2,JC)*YJI + U(3,JC)*ZJI)/EL
      Q1 = (U(1,IC)*XJI + U(2,IC)*YJI + U(3,IC)*ZJI)/EL
      EPSXX = (Q2 - Q1)/EL
      SIGMA = EPSXX*EE
      FAX = SIGMA*RA
      WRITE(6,26)IC,JC,RA,EE,EL,FAX,SIGMA
C**   ROD NO. 2
      Q2 = (U(1,NC)*XNJ + U(2,NC)*YNJ + U(3,NC)*ZNJ)/EH
      Q1 = (U(1,JC)*XNJ + U(2,JC)*YNJ + U(3,JC)*ZNJ)/EH
      EPSXX = (Q2 - Q1)/EH
      SIGMA = EPSXX*EE
      FAX = SIGMA*RA
      WRITE(6,26)JC,NC,RA,EE,EH,FAX,SIGMA
C**   ROD NO. 3
      Q2 = (U(1,NC)*XNI + U(2,NC)*YNI + U(3,NC)*ZNI)/EK
      Q1 = (U(1,IC)*XNI + U(2,IC)*YNI + U(3,IC)*ZNI)/EK
      EPSXX = (Q2 - Q1)/EK
      SIGMA = EPSXX*EE
      FAX = SIGMA*RA
      WRITE(6,26)IC,NC,RA,EE,EK,FAX,SIGMA
   50 CONTINUE
   15 FORMAT(10X,3I4,4X,6(E10.4,4X))
   16 FORMAT('1',//,40X,'C.S.T. ELEMENT STRESSES AND STRAINS'//)
   17 FORMAT(10X,'NODAL PT.NO.S',3X,'STRAIN-XX',5X,'STRAIN-YY',5X,'STRAI
     1N-XY',5X,'STRESS-XX',5X,'STRESS-YY',5X,'SHEAR-XY',//)
   21 FORMAT(2X,'ROD NOD.PTS',3X,'   AREA    ',4X,'  E-VALUE  ',4X,'   LEN
     CGTH   ',4X,'AXIAL LOAD',4X,'  STRESS   ',/)
   25 FORMAT( )
   26 FORMAT(/,4X,2I4,4X,5(E10.4,4X))
   27 FORMAT(/,10X,'IDEALIZED TRIANGLE STIFFENER LOADS',/)
      RETURN
      END
```

```
      SUBROUTINE SHTRAP(NSHTR,IMATK,NODAL,A,XX,YY,ZZ,JID,NBAND)
C**   STIFFNESS MATRIX FOR QUADRILATERAL SHEAR AND IDEALIZED PANEL.
      DIMENSION S(8,8),ST(12,12),TR(12,12),TRT(12,12),STR(12,12)
      DIMENSION STF(12,12),NOD(4),A(IMATK,NBAND)
      DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL),AREA(2)
      N = 8
      IF(JID.NE.0) GO TO 10
```

```
      WRITE(6,5)
      GO TO 14
   10 WRITE(6,6)
   14 DO 50 ISTR = 1,NSHTR
      READ(5,15)IS,JS,MS,NS,T,ANU,EE
      WRITE(6,25)IS,JS,MS,NS,T,ANU,EE
      NOD(1) = IS
      NOD(2) = JS
      NOD(3) = MS
      NOD(4) = NS
C**   TRANSFORM THE ELEMENT INTO A LOCAL COORDINATE SYSTEM
      XJI = XX(JS) - XX(IS)
      XNI = XX(NS) - XX(IS)
      XMJ = XX(MS) - XX(JS)
      XMI = XX(MS) - XX(IS)
      XNJ = XX(NS) - XX(JS)
      YNJ = YY(NS) - YY(JS)
      YMI = YY(MS) - YY(IS)
      YMJ = YY(MS) - YY(JS)
      YNI = YY(NS) - YY(IS)
      YJI = YY(JS) - YY(IS)
      ZJI = ZZ(JS) - ZZ(IS)
      ZNI = ZZ(NS) - ZZ(IS)
      ZMJ = ZZ(MS) - ZZ(JS)
      ZMI = ZZ(MS) - ZZ(IS)
      ZNJ = ZZ(NS) - ZZ(JS)
C**   CALCULATE THE LENGTHS OF THE SIDES + DIAGONALS OF THE QUADRILATERAL
      EL = SQRT(XJI**2 + YJI**2 + ZJI**2)
      EH = SQRT(XNI**2 + YNI**2 + ZNI**2)
      EI = SQRT(XMJ**2 + YMJ**2 + ZMJ**2)
      IF(EL.GT.0.0.AND.EH.GT.0.0.AND.EI.GT.0.0) GO TO 7
      WRITE(6,35) ISTR
      CALL EXIT
    7 EK = SQRT(XMI**2 + YMI**2 +  ZMI**2)
      EM = SQRT(XNJ**2 + YNJ**2 +  ZNJ**2)
      EX = (EL**2 + EI**2- EK**2)/(2.0*EL)
      WHY = (EH**2 + EL**2-EM**2)/(2.0*EL)
      H1 = SQRT(EH**2 - WHY**2)
      H2 = SQRT(EI**2 - EX**2)
C**   DIMENSIONS IN LOCAL COORDINATES
      XBI = 0
      YBI = 0
      XBJ = EL
      YBJ = 0
      XBM = EL - EX
      YBM = H2
      XBN = WHY
      YBN = H1
      XBNI = XBN - XBI
      XBNJ = XBN - XBJ
      XBMI = XBM - XBI
      YBNI = YBN - YBI
      YBMI = YBM - YBI
      XBJI = XBJ - XBI
      YBNJ = YBN - YBJ
      AREA = XBJI*YBMI + XBMI*YBNJ   -XBNI*YBMI
      G = EE/(2.0*(1.0 + ANU))
      Z = T*G/(2.0*AREA)
      AA= (XBNJ**2)*Z
      B = YBNI*XBNJ*Z
```

```
      C = XBMI*XBNJ*Z
      D = YBMI*XBNJ*Z
      E = (YBNI**2)*Z
      F = YBNI*XBMI*Z
      GE = YBMI*YBNI*Z
      AI = (XBMI**2)*Z
      AJ = YBMI*XBMI*Z
      AL = (YBMI**2)*Z
      S(1,1) = AA
      S(5,1) = -AA
      S(5,5) = AA
      S(2,1) = -B
      S(6,1) = B
      S(5,2) = B
      S(6,5) = -B
      S(3,1) = -C
      S(7,1) = C
      S(5,3) = C
      S(7,5) = -C
      S(4,1) = D
      S(8,1) = -D
      S(5,4) = -D
      S(8,5) = D
      S(2,2) = E
      S(6,6) = E
      S(6,2) = -E
      S(3,2) = F
      S(7,2) = -F
      S(6,3) = -F
      S(7,6) = F
      S(4,2) = -GE
      S(8,2) = GE
      S(6,4) = GE
      S(8,6) = -GE
      S(3,3) = AI
      S(7,7) = AI
      S(7,3) = -AI
      S(4,3) = -AJ
      S(8,3) = AJ
      S(7,4) = AJ
      S(8,7) = -AJ
      S(4,4) = AL
      S(8,8) = AL
      S(8,4) = -AL
      NN = N-1
      DO 20 J = 1,NN
      IB = J + 1
      DO 20 I = IB,N
      S(J,I) = S(I,J)
   20 CONTINUE
      CALL PREPEL(8,S,ST)
      CALL PLATRA(IS,JS,NS,XX,YY,ZZ,NODAL,TR)
      CALL TRPOSE(TR,TRT,12,12)
      CALL AMTMUL(12,12,TRT,ST,STR,12)
      CALL AMTMUL(12,12,STR,TR,STF,12)
      CALL ELEPAC(12,STF,A,NOD,IMATK,NBAND)
      IF(JID.EQ.0) GO TO 50
C**   STIFFENERS FOR IDEALIZED PANEL
      A1 = H1*T/6.0
      A2 = H2*T/6.0
```

```
      A3 = EL*T/6.0
      A4 = (EL-EX-WHY)*T/6.0
      AREA(1) = (A1 + A2)/2.0
      AREA(2) = (A3 + A4)/2.0
C**  ROD NO. 1
      RA = AREA(1)
      CALL RODEL3(RA,EE,XX,YY,ZZ,IS,JS,STF,NODAL)
      NOD(1) = IS
      NOD(2) = JS
      CALL ELEPAC(6,STF,A,NOD,IMATK,NBAND)
C**  ROD NO. 2
      CALL RODEL3(RA,EE,XX,YY,ZZ,MS,NS,STF,NODAL)
      NOD(1) = MS
      NOD(2) = NS
      CALL ELEPAC(6,STF,A,NOD,IMATK,NBAND)
C**  ROD NO. 3
      RA = AREA(2)
      CALL RODEL3(RA,EE,XX,YY,ZZ,IS,NS,STF,NODAL)
      NOD(1) = IS
      NOD(2) = NS
      CALL ELEPAC(6,STF,A,NOD,IMATK,NBAND)
C**  ROD NO. 4
      CALL RODEL3(RA,EE,XX,YY,ZZ,JS,MS,STF,NODAL)
      NOD(1) = JS
      NOD(2) = MS
      CALL ELEPAC(6,STF,A,NOD,IMATK,NBAND)
   50 CONTINUE
    5 FORMAT(10X,'SHEAR PANEL DATA',/)
    6 FORMAT(10X,'IDEALIZED PANEL DATA',/)
   15 FORMAT( )
   25 FORMAT(10X,4I6,3E13.4)
   35 FORMAT(10X,'***ERROR*** INCORRECT NUMBERING OR NODAL POINT COORDIN
     1ATES OF SHEAR OR IDEALIZED PANEL, CARD NO.',I6//)
      RETURN
      END


      SUBROUTINE TRPOSE(A,ATR,IR,IC)
C**  SUBROUTINE TRANSPOSES THE A-MATRIX TO THE ATR-MATRIX
      DIMENSION A(IR,IC),ATR(IC,IR)
      DO 10 I = 1,IR
      DO 10 J = 1,IC
      ATR(J,I) = A(I,J)
   10 CONTINUE
      RETURN
      END


      SUBROUTINE  PREPEL(ORDER,S,SA)
C**  SUBROUTINE FILLS IN ROWS AND COLUMNS OF ZERO'S IN 2D MATRIX
C**  TO PREPARE IT FOR MULTIPLICATION WITH THE TRANSFORMATION MATRIX.
      INTEGER ORDER
      DIMENSION SA(12,12), TEMP(64)  ,S(ORDER,ORDER)
```

```
      NORDER = ORDER/2 + ORDER
C**   VACATE SA MATRIX
      DO 10 I = 1,NORDER
      DO 10 J = 1,NORDER
      SA(I,J) = 0
   10 CONTINUE
      IT = 0
      DO 20 J = 1,ORDER
      DO 20 I = 1,ORDER
      IT = IT + 1
      TEMP(IT) = S(I,J)
   20 CONTINUE
      IT = 0
      DO 50 J = 1,NORDER
      IF(MOD(J,3))25,50,25
   25 DO 40 I = 1,NORDER
      IF(MOD(I,3))30,40,30
   30 IT = IT + 1
      SA(I,J) = TEMP(IT)
   40 CONTINUE
   50 CONTINUE
      RETURN
      END

      SUBROUTINE ELEPAC(ORDER,STF,A,NOD,IMATK,NBAND)
      INTEGER ORDER
      DIMENSION A(IMATK,NBAND),STF(12,12),NOD(4)
      NORDER = ORDER/3
C**   ESTABLISH ROWS
      DO 350 JROW = 1,NORDER
      IR = NOD(JROW)
      NROWK = (IR-1)*3
      IF(NROWK) 350,305,305
  305 DO 350 JDI = 1,3
      NROWK = NROWK + 1
      IRSTF = (JROW-1)*3 + JDI
C**   ESTABLISH COLUMNS
      DO 330 KCOL = 1,NORDER
      IC = NOD(KCOL)
      NCOLK = (IC-1)*3
      DO 320 KDI = 1,3
      ICSTF = (KCOL - 1)*3 + KDI
      NCOL = NCOLK + KDI + 1 - NROWK
C**   SKIP STORING IF BELOW BAND
      IF (NCOL) 320,320,310
  310 A(NROWK,NCOL) = A(NROWK,NCOL) + STF(IRSTF,ICSTF)
  320 CONTINUE
  330 CONTINUE
  350 CONTINUE
      RETURN
      END

      SUBROUTINE PANSTR(NSHTR,U,NODAL,XX,YY,ZZ,JID)
C**   CALCULATION OF STRESS AND STRAIN IN A SHEAR QUADRILATERAL AND
C**   IDEALIZED PANEL ELEMENT
      DIMENSION NOD(4),U(3,NODAL),V(12,12),VT(12,12),T(12,12)
      DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL)
      DIMENSION A(5,5),B(5),X(5),AA(5,5),AREA(2)
      DO 50 KS = 1,NSHTR
      READ(5,25)IS,JS,MS,NS,TH,ANU,EE
      NOD(1) = IS
      NOD(2) = JS
      NOD(3) = MS
      NOD(4) = NS
```

```
      G = EE/(2.0*(1.0 + ANU))
C**   SET UP LOCAL COORDINATE SYSTEM
      XMN = XX(MS) - XX(NS)
      YMN = YY(MS) - YY(NS)
      ZMN = ZZ(MS) - ZZ(NS)
      XJI = XX(JS) - XX(IS)
      YJI = YY(JS) - YY(IS)
      ZJI = ZZ(JS) - ZZ(IS)
      XNI = XX(NS) - XX(IS)
      YNI = YY(NS) - YY(IS)
      ZNI = ZZ(NS) - ZZ(IS)
      XMJ = XX(MS) - XX(JS)
      YMJ = YY(MS) - YY(JS)
      ZMJ = ZZ(MS) - ZZ(JS)
      XMI = XX(MS) - XX(IS)
      YMI = YY(MS) - YY(IS)
      ZMI = ZZ(MS) - ZZ(IS)
      XNJ = XX(NS) - XX(JS)
      YNJ = YY(NS) - YY(JS)
      ZNJ = ZZ(NS) - ZZ(JS)
      EL = SQRT(XJI**2 + YJI**2 + ZJI**2)
      EH = SQRT(XNI**2 + YNI**2 + ZNI**2)
      EI = SQRT(XMJ**2 + YMJ**2 + ZMJ**2)
      EK = SQRT(XMI**2 + YMI**2 +  ZMI**2)
      EM = SQRT(XNJ**2 + YNJ**2 +  ZNJ**2)
      EJ = SQRT(XMN**2 + YMN**2 + ZMN**2)
      EX = (EL**2 + EI**2- EK**2)/(2.0*EL)
      WHY = (EH**2 + EL**2-EM**2)/(2.0*EL)
      H1 = SQRT(EH**2 - WHY**2)
      H2 = SQRT(EI**2 - EX**2)
      H3 = (H2 + H1)/2.0
      DO 10 JC = 1,4
      NK = NOD(JC)
      DO 10 K2 = 1,3
      V(K2,JC) = U(K2,NK)
   10 CONTINUE
      CALL PLATRA(IS,JS,NS,XX,YY,ZZ,NODAL,T)
      CALL AMTMUL(3,4,T,V,VT,12)
      DO 20 I = 1,5
      DO 20 J = 1,5
      A(I,J) = 0.0
   20 CONTINUE
      B(5) = VT(2,2)-VT(2,1)
      B(2) = VT(1,3)-VT(1,1)-(EL-EX)/EL*(VT(1,2)-VT(1,1))
      B(3) = VT(2,3)-VT(2,1)
      B(4) = VT(1,4)-VT(1,1)-WHY/EL*(VT(1,2)-VT(1,1))
      B(1) = VT(2,4)-VT(2,1)
      A(5,2) = -0.5*(EL**2)
      A(2,2) = H2*(EL-EX)
      A(3,2) = -0.5*(EL-EX)**2
      A(4,2) = WHY*H1
      A(1,2) = -0.5*(WHY**2)
      A(3,1) = H2
      A(1,1) = H1
      A(2,3) = -0.5*(H2**2)
      A(3,3) = (EL-EX)*H2
      A(4,3) = -0.5*(H1**2)
      A(1,3) = WHY*H1
      A(2,4) = H2
      A(4,4) = H1
```

```
      A(5,5) = EL
      A(3,5) = (EL-EX)
      A(1,5) = WHY
      CALL  SIMEQ(5,A,AA,B,X,5)
      EPSXY = X(4) + X(5)
      TAU = EPSXY*G
      WRITE(6,15)(NOD(I),I = 1,4),EL,H3,G,TH,EPSXY,TAU
      IF(JID.EQ.0) GO TO 50
      A1 = H1*TH/6.0
      A2 = H2*TH/6.0
      A3 = EL*TH/6.0
      A4 = (EL-EX-WHY)*TH/6.0
      AREA(1) = (A1 + A2)/2.0
      AREA(2) = (A3 + A4)/2.0
      WRITE(6,35)
      WRITE(6,21)
C**   ROD NO. 1
      Q2 = (U(1,JS)*XJI + U(2,JS)*YJI + U(3,JS)*ZJI)/EL
      Q1 = (U(1,IS)*XJI + U(2,IS)*YJI + U(3,IS)*ZJI)/EL
      EPSXX = (Q2 - Q1)/EL
      SIGMA = EPSXX*EE
      FAX = SIGMA*AREA(1)
      WRITE(6,26)IS,JS,AREA(1),EE,EL,FAX,SIGMA
C**   ROD NO. 2
      Q2 = (U(1,MS)*XMN + U(2,MS)*YMN + U(3,MS)*ZMN)/EJ
      Q1 = (U(1,NS)*XMN + U(2,NS)*YMN + U(3,NS)*ZMN)/EJ
      EPSXX = (Q2 - Q1)/EJ
      SIGMA = EPSXX*EE
      FAX = SIGMA*AREA(1)
      WRITE(6,26)NS,MS,AREA(1),EE,EJ,FAX,SIGMA
C**   ROD NO. 3
      Q2 = (U(1,MS)*XMJ + U(2,MS)*YMJ + U(3,MS)*ZMJ)/EI
      Q1 = (U(1,JS)*XMJ + U(2,JS)*YMJ + U(3,JS)*ZMJ)/EI
      EPSXX = (Q2 - Q1)/EI
      SIGMA = EPSXX*EE
      FAX = SIGMA*AREA(2)
      WRITE(6,26)JS,MS,AREA(2),EE,EI,FAX,SIGMA
C**   ROD NO. 4
      Q2 = (U(1,NS)*XNI + U(2,NS)*YNI + U(3,NS)*ZNI)/EH
      Q1 = (U(1,IS)*XNI + U(2,IS)*YNI + U(3,IS)*ZNI)/EH
      EPSXX = (Q2 - Q1)/EH
      SIGMA = EPSXX*EE
      FAX = SIGMA*AREA(2)
      WRITE(6,26)IS,NS,AREA(2),EE,EH,FAX,SIGMA
      WRITE (6,45)
   50 CONTINUE
   15 FORMAT(2X,4I5,6E13.4)
   21 FORMAT(2X,'ROD NOD.PTS',3X,'   AREA    ',4X,'  E-VALUE ',4X,'   LEN
     CGTH   ',4X,'AXIAL LOAD',4X,'  STRESS    ',/)
   25 FORMAT( )
   26 FORMAT(/,4X,2I4,4X,5(E10.4,4X))
   35 FORMAT(20X,'IDEALIZED PANEL STIFFENER LOADS',/)
   45 FORMAT(///)
      RETURN
      END
```

```
      SUBROUTINE  SIMEQ(NFN,ADUP,A,B,X,IMATK)
      DIMENSION  ADUP(IMATK,IMATK),A(IMATK,IMATK),B(IMATK)
      DIMENSION X(IMATK), Y(250)
C*
C**  GAUSS ELIMINATION SOLUTION OF EQUATIONS
C*
      DO 820 K=1,NFN
      A(1,K)=ADUP(1,K)
      IF(K.EQ.1) GO TO 820
      A(K,1) = ADUP(K,1)/A(1,1)
  820 CONTINUE
      DO 850 J=2,NFN
      DO 850 K=J,NFN
      TOT1=0
      NJ=J-1
      DO 830 IS=1,NJ
      IF(J.NE.IS) GO TO 825
      TOT1 = TOT1 + A(IS,K)
      GO TO 830
  825 TOT1 = TOT1 + A(J,IS)*A(IS,K)
  830 CONTINUE
      A(J,K)= ADUP(J,K)-TOT1
      TOT2=0
      DO 840 IS=1,NJ
      IF(K.NE.IS) GO TO 835
      TOT2 = TOT2 + A(IS,J)
      GO TO 840
  835 TOT2 = TOT2 + A(K,IS)*A(IS,J)
  840 CONTINUE
      IF (K.EQ.J) GO TO 850
       A(K,J)= (ADUP(K,J)-TOT2)/A(J,J)
  850 CONTINUE
      Y(1)= B(1)
      DO 880 K=2,NFN
      TOT3=0
      NK=K-1
      DO 870 IS=1,NK
      IF (K.NE.IS) GO TO 860
      TOT3 = TOT3 + Y(IS)
      GO TO 870
  860 TOT3=TOT3+  A(K,IS)*Y(IS)
  870 CONTINUE
      Y(K)= B(K)-TOT3
  880 CONTINUE
      X(NFN)=Y(NFN)/A(NFN,NFN)
      DO 900 K=2,NFN
      KK=(NFN+1)-K
      TOT4=0
      NKK=KK+1
      DO 890 IS=NKK,NFN
      TOT4=TOT4+ A(KK,IS)*X(IS)
  890 CONTINUE
      X(KK)= (Y(KK)-TOT4)/ A(KK,KK)
  900 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE AMTMUL(IMR,IMC,F,B,C,ISIZE)
      DIMENSION F(ISIZE,ISIZE),B(ISIZE,ISIZE),C(ISIZE,ISIZE)
C*
C**  TO MULTIPLY TWO MATRICES (F-COLS = B-ROWS)
C*
      DO 20 IC=1,IMR
      DO 20 J=1,IMC
      TOTROW=0
      DO 10 I=1,IMR
      TOTROW=TOTROW+F(IC,I)*B(I,J)
   10 CONTINUE
      C(IC,J)=TOTROW
   20 CONTINUE
      RETURN
      END

      SUBROUTINE  TRISH(NTRIS,IMATK,NODAL,A,XX,YY,ZZ,JID,NBAND)
C**  STIFFNESS MATRIX OF SHEAR OR IDEALIZED TRIANGLE PANEL
      DIMENSION SK(6,6),SKP(12,12),TR(12,12),TRT(12,12),SKTR(12,12)
      DIMENSION  NOD(3),A(IMATK,NBAND),SKF(12,12)
      DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL)
      N = 6
      IF(JID.EQ.1) GO TO 8
      WRITE(6,5)
      GO TO 14
    8 WRITE(6,9)
   14 DO 50 IST = 1,NTRIS
      READ(5,15) IS,JS,NS,TH,ANU,EE
      WRITE(6,25) IS,JS,NS,TH,ANU,EE
      NOD(1) = IS
      NOD(2) = JS
      NOD(3) = NS
      G = EE/(2.0*(1.0 + ANU))
C**  SET UP LOCAL COORDINATE SYSTEM
       XJI = XX(JS) - XX(IS)
       YJI = YY(JS) - YY(IS)
       ZJI = ZZ(JS) - ZZ(IS)
       XNI = XX(NS) - XX(IS)
       YNI = YY(NS) - YY(IS)
       ZNI = ZZ(NS) - ZZ(IS)
       XNJ = XX(NS) - XX(JS)
       YNJ = YY(NS) - YY(JS)
       ZNJ = ZZ(NS) - ZZ(JS)
       EL = SQRT(XJI**2 + YJI**2 + ZJI**2)
       EK = SQRT(XNI**2 + YNI**2 + ZNI**2)
       EH = SQRT(XNJ**2 + YNJ**2 + ZNJ**2)
       IF(EL.GT.0.0.AND.EK.GT.0.0.AND.EH.GT.0.0 ) GO TO 7
       WRITE(6,35) IST
       CALL EXIT
    7  WHY = (EK**2 + EL**2 - EH**2)/(2.0*EL)
       IF(WHY.GE.0.0.AND.WHY.LE.EL) GO TO 16
       WRITE(6,17) IST
       CALL EXIT
   16  H1 = SQRT(EK**2 - WHY**2)
       AREA = 0.5*EL*H1
       D = G*TH*AREA/(EL**2)
       SK(1,1) = ((WHY-EL)/H1)**2 *D
       SK(2,1) = -(WHY-EL)/H1*D
       SK(3,1) = -WHY*(WHY-EL)/(H1*H1)*D
       SK(4,1) = (WHY-EL)/H1*D
       SK(5,1) = EL*(WHY-EL)/(H1*H1)*D
       SK(6,1) = 0.0
       SK(2,2) = D
```

```
      SK(3,2) = WHY/H1*D
      SK(4,2) = -D
      SK(5,2) = -EL/H1*D
      SK(6,2) = 0.0
      SK(3,3) = ((WHY/H1)**2)*D
      SK(4,3) = -WHY/H1*D
      SK(5,3) = -WHY*EL/(H1*H1)*D
      SK(6,3) = 0.0
      SK(4,4) = D
      SK(5,4) = EL/H1*D
      SK(6,4) = 0.0
      SK(5,5) = ((EL/H1)**2)*D
      SK(6,5) = 0.0
      SK(6,6) = 0.0
      NN = N-1
      DO 10 J = 1,NN
      IB = J + 1
      DO 10 I = IB,N
      SK(J,I) = SK(I,J)
   10 CONTINUE
      CALL PREPEL(6,SK,SKP)
      CALL  PLATRA(IS,JS,NS,XX,YY,ZZ,NODAL,TR)
      CALL  TRPOSE(TR,TRT,12,12)
      CALL  AMTMUL(9,9,TRT,SKP,SKTR,12)
      CALL  AMTMUL(9,9,SKTR,TR,SKF,12)
      CALL  ELEPAC(9,SKF,A,NOD,IMATK,NBAND)
      IF(JID.EQ.0) GO TO 50
C**  IDEALIZED TRIANGLE  STIFFENERS ARE ADDED
      COST = WHY/EK
      COTT = WHY/H1
      TANT = 1.0/COTT
      SINT = H1/EK
      TANPH = H1/(EL-WHY)
      COSPH = (EL-WHY)/EH
      SINPH = H1/EH
      AC = 0.5*(EK/COST - EL)*COTT
      DC = 0.5*(EK*TANT - (EK/COST - EL)/SINT)
      BC = 0.5*(EH*TANPH - (EH/COSPH - EL)/SINPH)
      HDASH = 2./3.*AREA*(1./EL + 1./EK + 1./EH)
      RA = TH*HDASH**3/(12.0*(AC**2+DC**2+BC**2))
C**  ROD NO. 1
      CALL  RODEL3(RA,EE,XX,YY,ZZ,IS,JS,SKF,NODAL)
      NOD(1) = IS
      NOD(2) = JS
      CALL  ELEPAC(6,SKF,A,NOD,IMATK,NBAND)
C**  ROD NO. 2
      CALL RODEL3(RA,EE,XX,YY,ZZ,JS,NS,SKF,NODAL)
      NOD(1) = JS
      NOD(2) = NS
      CALL  ELEPAC(6,SKF,A,NOD,IMATK,NBAND)
C**  ROD NO. 3
      CALL RODEL3(RA,EE,XX,YY,ZZ,IS,NS,SKF,NODAL)
      NOD(1) = IS
      NOD(2) = NS
      CALL  ELEPAC(6,SKF,A,NOD,IMATK,NBAND)
   50 CONTINUE
    5 FORMAT(10X,'SHEAR TRIANGLE DATA',/)
    9 FORMAT(10X,'IDEALIZED TRIANGLE DATA',/)
   15 FORMAT( )
   17 FORMAT(10X,'*** ERROR *** BADLY SHAPED SHEAR TRIANGLE.CARD NO.',
     CI6,//)
   25 FORMAT(10X,3I6,3E13.4)
   35 FORMAT(10X,'***ERROR*** INCORRECT NUMBERING OR NODAL POINT COORDIN
     1ATES OF SHEAR TRIANGLE, CARD NO.',I6//)
      RETURN
      END
```

```
      SUBROUTINE RODPAK(NROD3,IMATK,NODAL,A,XX,YY,ZZ,NBAND)
C*
C**   SUBROUTINE READS DATA AND PACKS ENDLOAD ELEMENTS INTO STRUCTURE
C**   K-MATRIX
C*
      DIMENSION NOD(2),RODS(12,12), A(IMATK,NBAND)
      DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL)
      WRITE(6,14)
      DO 320 JROD = 1,NROD3
      READ(5,10)JR,KR,RA,RE
      WRITE(6,15)JR,KR,RA,RE
      CALL RODEL3 (RA,RE,XX,YY,ZZ,JR,KR,RODS,NODAL)
      NOD(1) = JR
      NOD(2) = KR
      CALL ELEPAC(6,RODS,A,NOD,IMATK,NBAND)
  320 CONTINUE
   10 FORMAT(  )
   14 FORMAT(10X,'FLANGE ELEMENT DATA',/)
   15 FORMAT(10X,2I6,2(E10.4,2X))
      RETURN
      END


      SUBROUTINE RODES3(NROD3,U,NODAL,XX,YY,ZZ)
C**   CALCULATION OF LOADS IN ENDLOAD ELEMENTS.
C*
C*
      DIMENSION U(3,NODAL),XX(NODAL),YY(NODAL),ZZ(NODAL)
      DO 960 JROD = 1,NROD3
      READ (5,10)JR,KR,RA,RE
      DIFX = XX(KR) - XX(JR)
      DIFY = YY(KR) - YY(JR)
      DIFZ = ZZ(KR) - ZZ(JR)
      EL = SQRT(DIFX**2 + DIFY**2 + DIFZ**2)
      Q2 = (U(1,KR)*DIFX + U(2,KR)*DIFY + U(3,KR)*DIFZ)/EL
      Q1 = (U(1,JR)*DIFX + U(2,JR)*DIFY + U(3,JR)*DIFZ)/EL
      EPSXX = (Q2-Q1)/EL
      SIGMA = EPSXX *RE
      FAX = SIGMA*RA
      WRITE(6,25)JR,KR,RA,RE,EL,FAX,SIGMA
  960 CONTINUE
   10 FORMAT( )
   25 FORMAT(/,4X,2I4,4X,5(E10.4,4X))
      RETURN
      END


      SUBROUTINE  QUATRI(XX,YY,ZZ,TH,EE,ANU,NODAL,SK,NOD)
C**   STIFFNESS MATRIX OF QUADRILATERAL ELEMENT MADE UP OF 4 CST ELEMENTS
      DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL)
      DIMENSION SK(10,10),EMAT(6,6)
      DIMENSION B(6,6),NOD(4),X(5),Y(5),LNOD(12)
      DIMENSION ITNOD(3),BTR(6,6),BE(6,6),S(36)
      I = NOD(1)
      J = NOD(2)
      M = NOD(3)
      N = NOD(4)
C**   SET UP LOCAL COORDINATE SYSTEM
      XJI = XX(J) - XX(I)
      YJI = YY(J) - YY(I)
```

```
      ZJI = ZZ(J) - ZZ(I)
      XNI = XX(N) - XX(I)
      YNI = YY(N) - YY(I)
      ZNI = ZZ(N) - ZZ(I)
      XMI = XX(M) - XX(I)
      YMI = YY(M) - YY(I)
      ZMI = ZZ(M) - ZZ(I)
      XNJ = XX(N) - XX(J)
      YNJ = YY(N) - YY(J)
      ZNJ = ZZ(N) - ZZ(J)
      XMJ = XX(M) - XX(J)
      YMJ = YY(M) - YY(J)
      ZMJ = ZZ(M) - ZZ(J)
      EL = SQRT(XJI**2+YJI**2+ZJI**2)
      EH = SQRT(XNI**2+YNI**2+ZNI**2)
      EI = SQRT(XMJ**2+YMJ**2+ZMJ**2)
      IF(EL.GT.0.0.AND.EH.GT.0.0.AND.EI.GT.0.0) GO TO 4
      WRITE(6,35)
      CALL EXIT
    4 EK = SQRT(XMI**2+YMI**2+ZMI**2)
      EM = SQRT(XNJ**2+YNJ**2+ZNJ**2)
      EX = (EL**2 + EI**2 - EK**2)/(2.0*EL)
      WHY = (EH**2 + EL**2 - EM**2)/(2.0*EL)
      H1 = SQRT(EH**2 - WHY**2)
      H2 = SQRT(EI**2 - EX**2)
      X(1) = 0.0
      Y(1) = 0.0
      X(2) = EL
      Y(2) = 0.0
      X(3) = EL-EX
      Y(3) = H2
      X(4) = WHY
      Y(4) = H1
      X(5) = H1*EL*(EL-EX)/(H2*(EL-WHY) + H1*(EL-EX))
      Y(5) = H2*X(5)/(EL-EX)
      EMULT = EE/(1.0 - ANU**2)
      DO 5 IR = 1,10
      DO 5 JR = 1,10
    5 SK(IR,JR) = 0.0
      DO 10 IR = 1,6
      DO 10 JR = 1,6
   10 EMAT(IR,JR) = 0.0
C**   PLANE STRESS E-MATRIX
      EMAT(1,1) = EMULT
      EMAT(2,2) = EMULT
      EMAT(2,1) = EMULT*ANU
      EMAT(1,2) = EMULT*ANU
      EMAT(3,3) = (1.0-ANU)/2.0*EMULT
      LNOD(1) = 1
      LNOD(2) = 2
      LNOD(3) = 5
      LNOD(4) = 5
      LNOD(5) = 2
      LNOD(6) = 3
      LNOD(7) = 4
      LNOD(8) = 5
      LNOD(9) = 3
      LNOD(10) = 1
      LNOD(11) = 5
      LNOD(12) = 4
```

```
      ICOUNT = 0
      DO 200 ITR = 1,4
      IF(ITR.NE.1) GO TO 20
      ICOUNT = 1
      IEND = 3
      GO TO 30
   20 ICOUNT = ICOUNT + 3
      IEND = ICOUNT + 2
   30 JNOD = 0
      DO 40 ILD = ICOUNT,IEND
      JNOD = JNOD + 1
   40 ITNOD(JNOD) = LNOD(ILD)
      IS = ITNOD(1)
      JS = ITNOD(2)
      NS = ITNOD(3)
      A1 = X(JS)*Y(NS)+X(IS)*Y(JS)+Y(IS)*X(NS)
      A2 = X(JS)*Y(IS) + X(NS)*Y(JS) + Y(NS)*X(IS)
      AREA = 0.5*ABS(A1-A2)
      DO 50 IST = 1,6
      DO 50 JST = 1,6
   50 B(IST,JST) = 0.0
      B(1,1) = Y(JS) - Y(NS)
      B(1,3) = Y(NS) - Y(IS)
      B(1,5) = Y(IS) - Y(JS)
      B(2,2) = X(NS) - X(JS)
      B(2,4) = X(IS) - X(NS)
      B(2,6) = X(JS) - X(IS)
      B(3,1) = X(NS) - X(JS)
      B(3,2) = Y(JS) - Y(NS)
      B(3,3) = B(2,4)
      B(3,4) = B(1,3)
      B(3,5) = B(2,6)
      B(3,6) = B(1,5)
      DO 60 IST = 1,6
      DO 60 JST = 1,6
   60 B(IST,JST) = B(IST,JST)/(2.0*AREA)
      CALL  TRPOSE(B,BTR,6,6)
      CALL  AMTMUL(6,3,BTR,EMAT,BE,6)
      CALL  AMTMUL(6,6,BE,B,BTR,6)
      DO 70 KTR = 1,6
      DO 70 JTR = 1,6
   70 B(KTR,JTR) = BTR(KTR,JTR)*TH*AREA
C**   MAKE LOCAL MATRIX INTO A COLUMN
      IT = 0
      DO 80 J = 1,6
      DO 80 I = 1,6
      IT = IT + 1
   80 S(IT) = B(I,J)
C**   PACK LOCAL MATRIX INTO QUAD MATRIX
      IT = 0
      DO90 ICOL = 1,3
      IK = ITNOD(ICOL)
      DO 90 JDI = 1,2
      IACOL = (IK-1)*2 + JDI
      DO 90 IROW = 1,3
      IR = ITNOD(IROW)
      DO 90 IDI = 1,2
      IAROW = (IR-1)*2 + IDI
      IT = IT + 1
      SK(IAROW,IACOL) = SK(IAROW,IACOL) + S(IT)
```

```
   90 CONTINUE
  200 CONTINUE
   35 FORMAT(10X,'***ERROR*** INCORRECT NUMBERING OR NODAL POINT COORDIN
     1ATES OF 4-CST PANEL.',//)
      RETURN
      END

      SUBROUTINE  QUA4(NQUA4,IMATK,NODAL,A,XX,YY,ZZ,NBAND)
C**   STIFFNESS MATRIX OF 4-NODAL POINT QUADRILATERAL
C**   PLANE STRESS ELEMENT OF CONSTANT THICKNESS
C**   THE SIDES REMAIN STRAIGHT AFTER DEFORMATION
      DIMENSION E(3,3),SE(8,8)  ,ST(12,12),TR(12,12),TRT(12,12)
      DIMENSION XL(4),YL(4),B(3,8),NOD(4),SA(12,12)
      DIMENSION A(IMATK,NBAND),XX(NODAL),YY(NODAL),ZZ(NODAL)
      WRITE(6,5)
      DO 200 IQ = 1,NQUA4
      READ(5,15)IS,JS,MS,NS,TH,ANU,EE
      WRITE(6,25) IS,JS,MS,NS,TH,ANU,EE
      NOD(1) = IS
      NOD(2) = JS
      NOD(3) = MS
      NOD(4) = NS
      I = IS
      J = JS
      M = MS
      N = NS
C**   SET UP LOCAL COORDINATE SYSTEM .
      XJI = XX(J) - XX(I)
      YJI = YY(J) - YY(I)
      ZJI = ZZ(J) - ZZ(I)
      XNI = XX(N) - XX(I)
      YNI = YY(N) - YY(I)
      ZNI = ZZ(N) - ZZ(I)
      XMI = XX(M) - XX(I)
      YMI = YY(M) - YY(I)
      ZMI = ZZ(M) - ZZ(I)
      XNJ = XX(N) - XX(J)
      YNJ = YY(N) - YY(J)
      ZNJ = ZZ(N) - ZZ(J)
      XMJ = XX(M) - XX(J)
      YMJ = YY(M) - YY(J)
      ZMJ = ZZ(M) - ZZ(J)
      EL = SQRT(XJI**2+YJI**2+ZJI**2)
      EH = SQRT(XNI**2+YNI**2+ZNI**2)
      EI = SQRT(XMJ**2+YMJ**2+ZMJ**2)
      IF(EL.GT.0.0.AND.EH.GT.0.0.AND.EI.GT.0.0) GO TO 7
      WRITE(6,35)IQ
      CALL EXIT
    7 EK = SQRT(XMI**2+YMI**2+ZMI**2)
      EM = SQRT(XNJ**2+YNJ**2+ZNJ**2)
      EX = (EL**2 + EI**2 - EK**2)/(2.0*EL)
      WHY = (EH**2 + EL**2 - EM**2)/(2.0*EL)
      H1 = SQRT(EH**2 - WHY**2)
      H2 = SQRT(EI**2 - EX**2)
      XL(1) = 0.0
      YL(1) = 0.0
      XL(2) = EL
      YL(2) = 0.0
      XL(3) = EL-EX
      YL(3) = H2
      XL(4) = WHY
      YL(4) = H1
```

```
      DO 10 IE = 1,3
      DO 10 JE = 1,3
   10 E(IE,JE) = 0.0
C**  PLANE STRESS E-MATRIX
      EMULT = EE/(1.0-ANU**2)
      E(1,1) = EMULT
      E(2,2) = EMULT
      E(1,2) = EMULT*ANU
      E(2,1) = EMULT*ANU
      E(3,3) = (1.0 - ANU)/2.0*EMULT
C**  CLEAR ARRAYS.OVERWRITE MATRIX E BY UPPER TRIANGLE U.
      DO 3 K = 1,8
      DO 3 L = K,8
    3 SE(K,L) = 0.0
      E(1,1) = SQRT(E(1,1))
      E(1,2) = E(1,2)/E(1,1)
      E(1,3) = E(1,3)/E(1,1)
      E(2,2) = SQRT(E(2,2) - E(1,2)*E(1,2))
      E(2,3) = (E(2,3) - E(1,2)*E(1,3))/E(2,2)
      E(3,3) = SQRT(E(3,3)-E(1,3)*E(1,3)-E(2,3)*E(2,3))
C**  START GAUSS QUADRATURE LOOP.
      DO 20 II = 1,2
      DO 20 JJ = 1,2
      CALL  SHAPE(II,JJ,XL,YL,B,DETJAC)
C**  OVERWRITE MATRIX B BY U*B.
      DO 9 K = 1,3
      DO 9 L = 1,8
      DUM1 = 0.0
      DO 8 M = K,3
    8 DUM1 = DUM1 + E(K,M)*B(M,L)
    9 B(K,L) = DUM1
C**  ADD IN CONTRIBUTION TO ELEMENT MATRICES.
      DUM1 = DETJAC
      DO 20 NROW = 1,8
      DO 20 NCOL = NROW,8
      DUM2 = 0.0
      DO 18 L = 1,3
   18 DUM2 = DUM2 + B(L,NROW)*B(L,NCOL)
   20 SE(NROW,NCOL) = SE(NROW,NCOL) + DUM1*DUM2
C**  QUADRATURE ENDED.COMPLETE STIFFNESS MATRIX BY
C**  SYMMETRY AND MULTIPLY BY THE PANEL THICKNESS
      DO 30 K = 2,8
      DO 30 L = 1,K
   30 SE(K,L) = SE(L,K)
      DO 40 K = 1,8
      DO 40 L = 1,8
   40 SE(K,L) = TH*SE(K,L)
C**  PREPARE ,ROTATE AND PACK ELEMENT INTO STRUCTURE
C**  STIFFNESS MATRIX
      CALL  PREPEL(8,SE,ST)
      CALL  PLATRA(IS,JS,NS,XX,YY,ZZ,NODAL,TR)
      CALL  TRPOSE(TR,TRT,12,12)
      CALL  AMTMUL(12,12,TRT,ST,SA,12)
      CALL  AMTMUL(12,12,SA,TR,ST,12)
      CALL  ELEPAC(12,ST,A,NOD,IMATK,NBAND)
  200 CONTINUE
    5 FORMAT(10X,'QUA4 PANEL DATA',/)
   15 FORMAT( )
   25 FORMAT(10X,4I6,3E13.4)
   35 FORMAT(10X,'***ERROR*** INCORRECT NUMBERING OR NODAL POINT COORDIN
```

```
     1ATES OF QUA4 ELEMENT,CARD NO.',I6//)
      RETURN
      END


      SUBROUTINE RODEL3(RA,RE,XX,YY,ZZ,JR,KR,RODS,NODAL)
      DIMENSION RODS(12,12),XX(NODAL),YY(NODAL),ZZ(NODAL)
C*
C**  CALCULATION OF END-LOAD ELEMENT STIFFNESSES IN 3-DIM.
C*
      N = 6
      DIFX = XX(KR) - XX(JR)
      DIFY = YY(KR) - YY(JR)
      DIFZ = ZZ(KR) - ZZ(JR)
      EL = SQRT(DIFX**2 + DIFY**2 + DIFZ**2)
      IF (EL.GT.0.0) GO TO 7
      WRITE(6,35)
      CALL EXIT
    7 RO = SQRT(DIFZ**2 + DIFX**2)
C**  TRANSFORMATION COEFFICIENTS
      Z = RA*RE/EL
      T11 = DIFX/EL
      T12 = DIFY/EL
      T13 = DIFZ/EL
      A = T11*T11*Z
      B = T11*T12*Z
      C = T11*T13*Z
      D = T12*T12*Z
      E = T12*T13*Z
      F = T13*T13*Z
      RODS(1,1) = A
      RODS(2,1) = B
      RODS(3,1) = C
      RODS(4,1) = -A
      RODS(5,1) = -B
      RODS(6,1) = -C
      RODS(2,2) = D
      RODS(3,2) = E
      RODS(4,2) = -B
      RODS(5,2) = -D
      RODS(6,2) = -E
      RODS(3,3) = F
      RODS(4,3) = -C
      RODS(5,3) = -E
      RODS(6,3) = -F
      RODS(4,4) = A
      RODS(5,4) = B
      RODS(6,4) = C
      RODS(5,5) = D
      RODS(6,5) = E
      RODS(6,6) = F
      N1 = N-1
      DO 20 J = 1,N1
      IB = J + 1
      DO 20 I = IB,N
      RODS(J,I) = RODS(I,J)
   20 CONTINUE
   35 FORMAT(10X,'***ERROR*** AN ENDLOAD ELEMENT HAS NO LENGTH.CHECK NOD
     1AL POINT NUMBERING AND COORDINATES',//)
      RETURN
      END
```

```
      SUBROUTINE SUPORT(A,B,IMATK,NBAND,IDISP)
C**   INSERTION OF BOUNDARY CONDITIONS
      DIMENSION A(IMATK,NBAND),B(IMATK)
      DO 500 IDIS = 1,IDISP
      READ(5,50)INOD,ID,DISP
      WRITE(6,515) INOD,ID,DISP
      NROWK = (INOD - 1)*3 + ID
      A(NROWK,1) = 1.0
      B(NROWK) = DISP
      DO 430 J = 2,NBAND
      NR = NROWK + 1 - J
      NC = NROWK-1+J
      B(NR) = B(NR) - DISP*A(NR,J)
      B(NC) = B(NC) - DISP*A(NROWK,J)
      A(NROWK,J) = 0.0
      IF(NR)430,430,425
  425 A(NR,J) = 0.0
  430 CONTINUE
  500 CONTINUE
   50 FORMAT(  )
  515 FORMAT(10X,2I6,E10.4)
      RETURN
      END
```

```
      SUBROUTINE SOLVE(A,B,NN,IMATK,NBAND)
C**   GAUSS ELIMINATION SOLUTION OF EQUATIONS (BANDED COEFFICIENT MATRIX)
      DIMENSION A(IMATK,NBAND),B(IMATK)
C**   REDUCE MATRIX
      DO 300 N = 1,NN
      I = N
      DO 290 L = 2,NBAND
      I = I + 1
      IF(A(N,L)) 240,290,240
  240 C = A(N,L)/A(N,1)
      J = 0
      DO 270 K = L,NBAND
      J = J + 1
      IF(A(N,K)) 260,270,260
  260 A(I,J) = A(I,J) - C*A(N,K)
  270 CONTINUE
  280 A(N,L) = C
C**     AND LOAD VECTOR
C**     FOR EACH EQUATION
      B(I) = B(I) - C*B(N)
  290 CONTINUE
  300 B(N) = B(N)/A(N,1)
C**   BACK SUBSTITUTION
      N = NN
  350 N = N-1
      IF(N) 500,500,360
  360 L = N
      DO 400 K = 2,NBAND
      L = L + 1
      IF(A(N,K)) 370,400,370
  370 B(N) = B(N) - A(N,K)*B(L)
  400 CONTINUE
      GO TO 350
  500 RETURN
      END
```

```
      SUBROUTINE  QUAPAC(NOAT,IMATK,NODAL,A,XX,YY,ZZ,NBAND)
C**   STIFFNESS MATRIX FOR 4-CST QUADRILAT(PLANE STRESS)
      DIMENSION SK(10,10),S(8,8),ST(12,12),TR(12,12)
      DIMENSION TRT(12,12),STR(12,12),STF(12,12),NOD(4)
      DIMENSION A(IMATK,NBAND),XX(NODAL),YY(NODAL),ZZ(NODAL)
      DIMENSION Q10(8),Q9(8)
      WRITE(6,5)
      DO 100 IQT = 1,NQAT
      READ(5,15)IS,JS,MS,NS,TH,ANU,EE
      WRITE(6,25)IS,JS,MS,NS,TH,ANU,EE
      NOD(1) = IS
      NOD(2) = JS
      NOD(3) = MS
      NOD(4) = NS
      CALL  QUATRI(XX,YY,ZZ,TH,EE,ANU,NODAL,SK,NOD)
C**   REDUCE SK MATRIX TO AN 8X8
      D1 = SK(10,9)/SK(10,10) - SK(9,9)/SK(9,10)
      D2 = SK(10,10)/SK(10,9) - SK(9,10)/SK(9,9)
      DO 50 I3 = 1,8
      Q10(I3) = SK(9,I3)/(SK(9,9)*D2) - SK(10,I3)/(SK(10,9)*D2)
      Q9(I3) = SK(9,I3)/(SK(9,10)*D1) - SK(10,I3)/(SK(10,10)*D1)
   50 CONTINUE
      DO 70 IROW = 1,8
      DO 70 ICOL = 1,8
      S(IROW,ICOL)=SK(IROW,ICOL)+Q9(ICOL)*SK(IROW,9)+Q10(ICOL)*
     CSK(IROW,10)
   70 CONTINUE
      CALL  PREPEL(8,S,ST)
      CALL  PLATRA(IS,JS,NS,XX,YY,ZZ,NODAL,TR)
      CALL  TRPOSE(TR,TRT,12,12)
      CALL  AMTMUL(12,12,TRT,ST,STR,12)
      CALL  AMTMUL(12,12,STR,TR,STF,12)
      CALL  ELEPAC(12,STF,A,NOD,IMATK,NBAND)
  100 CONTINUE
    5 FORMAT(10X,'4-CST PANEL DATA',/)
   15 FORMAT(  )
   25 FORMAT(10X,4I6,3E13.4)
      RETURN
      END
```

```
      SUBROUTINE PLATRA(IS,JS,NS,XX,YY,ZZ,NODAL,T)
C**   PLANE ELEMENT TRANSFORMATION MATRIX(VECTOR METHOD)
C**   LOCAL COORDINATE SYSTEM SET UP WITH I AND J ON THE LOCAL X-AXIS
      DIMENSION T(12,12),XX(NODAL),YY(NODAL),ZZ(NODAL)
      XNI = XX(NS) - XX(IS)
      YNI = YY(NS) - YY(IS)
      ZNI = ZZ(NS) - ZZ(IS)
      XJI = XX(JS) - XX(IS)
      YJI = YY(JS) - YY(IS)
      ZJI = ZZ(JS) - ZZ(IS)
```

```
      XNJ = XX(NS) - XX(JS)
      YNJ = YY(NS) - YY(JS)
      ZNJ = ZZ(NS) - ZZ(JS)
      EL = SQRT(XJI**2 + YJI**2 + ZJI**2)
      EK = SQRT(XNI**2 + YNI**2 + ZNI**2)
      EH = SQRT(XNJ**2 + YNJ**2 + ZNJ**2)
      WHY = (EK**2 + EL**2 - EH**2)/(2.0*EL)
      H1 = SQRT(EK**2 - WHY**2)
      AREA = 0.5*EL*H1
      A = XJI/EL
      B = YJI/EL
      C = ZJI/EL
      G = (YJI*ZNI - ZJI*YNI)/(2.0*AREA)
      H = (ZJI*XNI - XJI*ZNI)/(2.0*AREA)
      P = (XJI*YNI - YJI*XNI)/(2.0*AREA)
      D = H*C - P*B
      E = P*A - G*C
      F = G*B - A*H
      DO 10 I = 1,12
      DO 10 J = 1,12
      T(I,J) = 0
   10 CONTINUE
      T(1,1) = A
      T(1,2) = B
      T(1,3) = C
      T(2,1) = D
      T(2,2) = E
      T(2,3) = F
      T(3,1) = G
      T(3,2) = H
      T(3,3) = P
      T(4,4) = A
      T(4,5) = B
      T(4,6 ) = C
      T(5,4) = D
      T(5,5) = E
      T(5,6) = F
      T(6,4) = G
      T(6,5) = H
      T(6,6) = P
      T(7,7) = A
      T(7,8) = B
      T(7,9) = C
      T(8,7) = D
      T(8,8) = E
      T(8,9) = F
      T(9,7) = G
      T(9,8) = H
      T(9,9) = P
      T(10,10) = A
      T(10,11) = B
      T(10,12) = C
      T(11,10) = D
      T(11,11) = E
      T(11,12) = F
      T(12,10) = G
      T(12,11) = H
      T(12,12) = P
      RETURN
      END
```

```
      SUBROUTINE QUASTR(NQAT,U,NODAL,XX,YY,ZZ)
C**   STRESSES IN THE 4-CST PANEL (PLANE STRESS)
      DIMENSION NOD(4),U(3,NODAL),V(12,12),VT(12,12)
      DIMENSION T(12,12),XX(NODAL),YY(NODAL),ZZ(NODAL)
      DIMENSION EPSX(4),EPSY(4),EPSXY(4),SK(10,10),DUP(8)
      DIMENSION SIGX(4),SIGY(4),SIGXY(4)
      WRITE(6,60)
      DO 100 IQT = 1,NQAT
      READ(5,15)IS,JS,MS,NS,TH,ANU,EE
      NOD(1) = IS
      NOD(2) = JS
      NOD(3) = MS
      NOD(4) = NS
C**   SET UP LOCAL COORDINATE SYSTEM
      XMN = XX(MS) - XX(NS)
      YMN = YY(MS) - YY(NS)
      ZMN = ZZ(MS) - ZZ(NS)
      XJI = XX(JS) - XX(IS)
      YJI = YY(JS) - YY(IS)
      ZJI = ZZ(JS) - ZZ(IS)
      XNI = XX(NS) - XX(IS)
      YNI = YY(NS) - YY(IS)
      ZNI = ZZ(NS) - ZZ(IS)
      XMJ = XX(MS) - XX(JS)
      YMJ = YY(MS) - YY(JS)
```

```
      ZMJ = ZZ(MS) - ZZ(JS)
      XMI = XX(MS) - XX(IS)
      YMI = YY(MS) - YY(IS)
      ZMI = ZZ(MS) - ZZ(IS)
      XNJ = XX(NS) - XX(JS)
      YNJ = YY(NS) - YY(JS)
      ZNJ = ZZ(NS) - ZZ(JS)
      EL = SQRT(XJI**2 + YJI**2 + ZJI**2)
      EH = SQRT(XNI**2 + YNI**2 + ZNI**2)
      EI = SQRT(XMJ**2 + YMJ**2 + ZMJ**2)
      EK = SQRT(XMI**2 + YMI**2 +  ZMI**2)
      EM = SQRT(XNJ**2 + YNJ**2 +  ZNJ**2)
      EJ = SQRT(XMN**2 + YMN**2 + ZMN**2)
      EX = (EL**2 + EI**2- EK**2)/(2.0*EL)
      WHY = (EH**2 + EL**2-EM**2)/(2.0*EL)
      H1 = SQRT(EH**2 - WHY**2)
      H2 = SQRT(EI**2 - EX**2)
      XO   = H1*EL*(EL-EX)/(H2*(EL-WHY) + H1*(EL-EX))
      YO   = H2*XO  /(EL-EX)
      DO 10 JC = 1,4
      NK = NOD(JC)
      DO 10 K2 = 1,3
      V(K2,JC) = U(K2,NK)
   10 CONTINUE
      CALL PQATRA(IS,JS,NS,XX,YY,ZZ,NODAL,T)
      CALL AMTMUL(3,4,T,V,VT,12)
      CALL QUATRI(XX,YY,ZZ,TH,EE,ANU,NODAL,SK,NOD)
C**  CALCULATION OF THE ELIMINATED POINT'S DEFLECTION
      D1 = SK(10,9)/SK(10,10) - SK(9,9)/SK(9,10)
      D2 = SK(10,10)/SK(10,9) - SK(9,10)/SK(9,9)
      IT = 0
      DO 20 IC = 1,4
      DO 20 IR = 1,2
      IT = IT + 1
      DUP(IT) = VT(IR,IC)
   20 CONTINUE
      TOT1 = 0
      TOT2 = 0
      DO 30 I1 = 1,8
      TOT1 = TOT1 + SK(9,I1)*DUP(I1)
      TOT2 = TOT2 + SK(10,I1)*DUP(I1)
   30 CONTINUE
      Q10 = (TOT1/SK(9,9) - TOT2/SK(10,9))/D2
      Q9 = (TOT1/SK(9,10) - TOT2/SK(10,10))/D1
C**  FIRST TRIANGLE
      AREA = 0.5*EL*YO
      EPSX(1) = (-YO*DUP(1) + YO*DUP(3))/(2.0*AREA)
      EPSY(1) = ((XO-EL)*DUP(2)-XO*DUP(4)+EL*Q10)/(2.0*AREA)
      EPSXY(1) = ((XO-EL)*DUP(1)-YO*DUP(2)-XO*DUP(3)+YO*DUP(4)
     C+EL*Q9)/(2.0*AREA)
C**  SECOND TRIANGLE
      A1 = EL*H2 + YO*(EL-EX)
      A2 = EX*YO + H2*XO
      AREA = 0.5*ABS(A1-A2)
      EPSX(2) = (-H2*Q9 + (H2-YO)*DUP(3)+YO*DUP(5))/(2.0*AREA)
      EPSY(2) = (-EX*Q10-(XO-(EL-EX))*DUP(4)+(EL-XO)*DUP(6))/(2.*AREA)
      EPSXY(2) = (-EX*Q9-H2*Q10+(XO-(EL-EX))*DUP(3)+(H2-YO)*DUP(4)
     C +(EL-XO)*DUP(5) + YO*DUP(6))/(2.0*AREA)
```

```
C**  THIRD TRIANGLE
      A1 = XO*H2 + WHY*YO + H1*(EL-EX)
      A2 = XO*H1 + (EL-EX)*YO + H2*WHY
      AREA = 0.5*ABS(A1 - A2)
      EPSX(3) = ((YO-H2)*DUP(7)+(H2-H1)*Q9+(H1-YO)*DUP(5))/(2.0*AREA)
      EPSY(3)=(((EL-EX)-XO)*DUP(8)+(WHY-(EL-EX))*Q10+(XO-WHY)*DUP(6))
     C/(2.0*AREA)
      EPSXY(3) = (((EL-EX)-XO)*DUP(7)+(YO-H2)*DUP(8)+(WHY-(EL-EX))*Q9
     C+(H2-H1)*Q10+(XO-WHY)*DUP(5)+(H1-YO)*DUP(6))/(2.0*AREA)
C**  FOURTH TRIANGLE
      A1 = XO*H1
      A2 = WHY*YO
      AREA = 0.5*ABS(A1-A2)
      EPSX(4) = ((YO-H1)*DUP(1)+H1*Q9-YO*DUP(7))/(2.0*AREA)
      EPSY(4) = ((WHY-XO)*DUP(2)-WHY*Q10+XO*DUP(8))/(2.0*AREA)
      EPSXY(4) = ((WHY-XO)*DUP(1)+(YO-H1)*DUP(2)-WHY*Q9+H1*Q10
     C+XO*DUP(7) - YO*DUP(8))/(2.0*AREA)
      TX = 0
      TY = 0
      TXY= 0
      DO 40 ITR = 1,4
      SIGX(ITR) = EE/(1.0-ANU**2)*(EPSX(ITR)+ANU*EPSY(ITR))
      SIGY(ITR) = EE/(1.0-ANU**2)*(EPSY(ITR)+ANU*EPSX(ITR))
      TX = TX + SIGX(ITR)
      TY = TY + SIGY(ITR)
      SIGXY(ITR) = EE/(2.0*(1.0 + ANU))*EPSXY(ITR)
      TXY = TXY + SIGXY(ITR)
   40 CONTINUE
      AVSIGX = TX/4.0
      AVSIGY = TY/4.0
      AVSXY = TXY/4.0
      WRITE(6,200) IS,JS,MS,NS
      WRITE(6,250)
      WRITE(6,400)(I,EPSX(I),EPSY(I),EPSXY(I),SIGX(I),SIGY(I),SIGXY(I),
     CI = 1,4)
      WRITE(6,300) AVSIGX,AVSIGY,AVSXY
  100 CONTINUE
   15 FORMAT( )
   60 FORMAT('1',///,30X,'4-CST  PANEL   STRESS RECOVERY',/,30X,30('*')
     C,//)
  200 FORMAT(//,4X,'ELEMENT CONN.PTS',4I5,/)
  250 FORMAT(2X,'TRIANGLE',6X,'EPSX',8X,'EPSY',10X,'EPSXY',8X,'SIGX',8X,
     C'SIGY',5X,'SIGXY',/)
  300 FORMAT(5X,' AVSIGX= ',E13.4,' AVSIGY= ',E13.4,' AVSIGXY= ',E13.4)
  400 FORMAT(5X,I4,6E13.4,/)
      RETURN
      END


      SUBROUTINE TRIM3(NTRIM,IMATK,NODAL,A,XX,YY,ZZ,NBAND,JID)
C**  STIFFNESS OF CST ELEMENT (PLANE STRESS OR PLANE STRAIN)
      DIMENSION EMAT(6,6),B(6,6),BTR(6,6),BE(6,6)
      DIMENSION CSP(12,12),TR(12,12),TRT(12,12),CSTR(12,12)
      DIMENSION NOD(4),A(IMATK,NBAND),CSF(12,12)
      DIMENSION XX(NODAL),YY(NODAL),ZZ(NODAL)
      IF(JID.EQ.1) GO TO 7
      WRITE(6,5)
      GO TO 14
    7 WRITE(6,8)
   14 DO 50 ICST = 1,NTRIM
      READ(5,15)IC,JC,NC,TH,ANU,EE
      WRITE(6,25)IC,JC,NC,TH,ANU,EE
      NOD(1) = IC
```

```
      NOD(2) = JC
      NOD(3) = NC
C**  SET UP LOCAL COORDINATE SYSTEM
      XJI = XX(JC) - XX(IC)
      XNI = XX(NC) - XX(IC)
      XNJ = XX(NC) - XX(JC)
      YJI = YY(JC) - YY(IC)
      YNI = YY(NC) - YY(IC)
      YNJ = YY(NC) - YY(JC)
      ZJI = ZZ(JC) - ZZ(IC)
      ZNI = ZZ(NC) - ZZ(IC)
      ZNJ = ZZ(NC) - ZZ(JC)
      EL = SQRT(XJI**2 + YJI**2 + ZJI**2)
      EK = SQRT(XNI**2 + YNI**2 + ZNI**2)
      EH = SQRT(XNJ**2 + YNJ**2 + ZNJ**2)
      IF(EL.GT.0.0.AND.EK.GT.0.0.AND.EH.GT.0.0) GO TO 13
      WRITE(6,35) ICST
      CALL EXIT
   13 WHY = (EK**2 + EL**2-EH**2)/(2.0*EL)
      IF(WHY.GE.0.0.AND.WHY.LE.EL) GO TO 16
      WRITE (6,17) ICST
      CALL EXIT
   16 H1 = SQRT(EK**2 - WHY**2)
      AREA = 0.5*EL*H1
      DO 10 IR = 1,6
      DO 10 IC = 1,6
      EMAT(IR,IC) = 0.0
   10 B(IR,IC) = 0.0
      IF(JID.EQ.1) GO TO 20
C**  PLANE STRAIN E-MATRIX
      EMULT = EE*(1.0 - ANU)/((1.0 + ANU)*(1.0 - 2.0*ANU))
      EMAT(1,1) = EMULT
      EMAT(2,1) = EMULT*ANU/(1.0 - ANU)
      EMAT(1,2) = EMULT*ANU/(1.0 - ANU)
      EMAT(2,2) = EMULT
      EMAT(3,3) = EMULT*(1.0 - 2.0*ANU)/(2.0*(1.0 - ANU))
      GO TO 24
C**  PLANE STRESS E-MATRIX
   20 EMULT = EE/(1.0 - ANU**2)
      EMAT(1,1) = EMULT
      EMAT(2,2) = EMULT
      EMAT(1,2) = EMULT*ANU
      EMAT(2,1) = EMULT*ANU
      EMAT(3,3) = (1.0 - ANU)/2.0*EMULT
   24 B(1,1) = -H1
      B(1,3) =  H1
      B(2,2) = WHY-EL
      B(2,4) = -WHY
      B(2,6) = EL
      B(3,1) = WHY - EL
      B(3,2) = -H1
      B(3,3) = -WHY
      B(3,4) = H1
      B(3,5) = EL
      DO 30 IR = 1,6
      DO 30 IC = 1,6
   30 B(IR,IC) = B(IR,IC)/(2.0*AREA)
      CALL TRPOSE(B,BTR,6,6)
      CALL AMTMUL(6,3,BTR,EMAT,BE,6)
      CALL AMTMUL(6,6,BE,B,BTR,6)
```

```
      DO 40 ITR = 1,6
      DO 40 JTR = 1,6
   40 B(ITR,JTR) = BTR(ITR,JTR)*TH*AREA
      CALL PREPEL(6,B,CSP)
      CALL PLATRA(IC,JC,NC,XX,YY,ZZ,NODAL,TR)
      CALL TRPOSE(TR,TRT,12,12)
      CALL AMTMUL(9,9,TRT,CSP,CSTR,12)
      CALL AMTMUL(9,9,CSTR,TR,CSF,12)
      CALL ELEPAC(9,CSF,A,NOD,IMATK,NBAND)
   50 CONTINUE
    5 FORMAT(10X,'C.S.T. DATA (PLANE STRAIN)',/)
    8 FORMAT(10X,'C.S.T.DATA (PLANE STRESS)',/)
   15 FORMAT( )
   17 FORMAT(10X,'***ERROR*** BADLY SHAPED CST ELEMENT.CARD NO.',I6//)
   25 FORMAT(10X,3I6,3E13.4)
   35 FORMAT(10X,'***ERROR*** INCORRECT NUMBERING OR NODAL-POINT COORDIN
     1ATES OF CST ELEMENT ,CARD NO.',I6//)
      RETURN
      END
```

```
      SUBROUTINE  Q4ST(NQUA4,U,NODAL,XX,YY,ZZ)
C**  STRESS CALCULATION IN THE QUA4 ELEMENT
      DIMENSION NOD(4),U(3,NODAL),V(12,12),VT(12,12)
      DIMENSION T(12,12),XX(NODAL),YY(NODAL),ZZ(NODAL)
      DIMENSION XL(4),YL(4),DUP(8)
      WRITE(6,60)
      WRITE(6,5)
      DO 200 IQ = 1,NQUA4
      READ(5,15)I,J,M,N,TH,ANU,EE
      NOD(1) = I
      NOD(2) = J
      NOD(3) = M
      NOD(4) = N
C**  SET UP LOCAL COORDINATE SYSTEM
      XJI = XX(J) - XX(I)
```

```
      YJI = YY(J) - YY(I)
      ZJI = ZZ(J) - ZZ(I)
      XNI = XX(N) - XX(I)
      YNI = YY(N) - YY(I)
      ZNI = ZZ(N) - ZZ(I)
      XMI = XX(M) - XX(I)
      YMI = YY(M) - YY(I)
      ZMI = ZZ(M) - ZZ(I)
      XNJ = XX(N) - XX(J)
      YNJ = YY(N) - YY(J)
      ZNJ = ZZ(N) - ZZ(J)
      XMJ = XX(M) - XX(J)
      YMJ = YY(M) - YY(J)
      ZMJ = ZZ(M) - ZZ(J)
      EL = SQRT(XJI**2+YJI**2+ZJI**2)
      EH = SQRT(XNI**2+YNI**2+ZNI**2)
      EI = SQRT(XMJ**2+YMJ**2+ZMJ**2)
      EK = SQRT(XMI**2+YMI**2+ZMI**2)
      EM = SQRT(XNJ**2+YNJ**2+ZNJ**2)
      EX = (EL**2 + EI**2 - EK**2)/(2.0*EL)
      WHY = (EH**2 + EL**2 - EM**2)/(2.0*EL)
      H1 = SQRT(EH**2 - WHY**2)
      H2 = SQRT(EI**2 - EX**2)
      XL(1) = 0.0
      YL(1) = 0.0
      XL(2) = EL
      YL(2) = 0.0
      XL(3) = EL-EX
      YL(3) = H2
      XL(4) = WHY
      YL(4) = H1
      DO 10 JC = 1,4
      NK = NOD(JC)
      DO 10 K2 = 1,3
   10 V(K2,JC) = U(K2,NK)
C**   TRANSFORM DEFLECTIONS-GLOBAL TO LOCAL
      CALL PLATRA(I,J,N,XX,YY,ZZ,NODAL,T)
      CALL AMTMUL(3,4,T,V,VT,12)
      IT = 0
      DO 20 IC = 1,4
      DO 20 IR = 1,2
      IT = IT + 1
   20 DUP(IT) = VT(IR,IC)
      AJ11 = 0.25*(XL(2) + XL(3) - XL(4))
      AJ21 = 0.25*(-XL(2) + XL(3) + XL(4))
      AJ12 = 0.25*(YL(3) - YL(4))
      AJ22 = 0.25*(YL(3) + YL(4))
      DETJ = AJ11*AJ22-AJ12*AJ21
      AJS11 = AJ22/DETJ
      AJS12 = -AJ12/DETJ
      AJS21 = -AJ21/DETJ
      AJS22 = AJ11/DETJ
      UXI = 0.25*(-DUP(1) + DUP(3) + DUP(5) - DUP(7))
      UET = 0.25*(-DUP(1) - DUP(3) + DUP(5) + DUP(7))
      VXI = 0.25*(-DUP(2) + DUP(4) + DUP(6) - DUP(8))
      VET = 0.25*(-DUP(2) - DUP(4) + DUP(6) + DUP(8))
      UX = AJS11*UXI + AJS12*UET
      UY = AJS21*UXI + AJS22*UET
      VX = AJS11*VXI + AJS12*VET
      VY = AJS21*VXI + AJS22*VET
```

```
      SIGX = EE/(1.0 - ANU**2)*(UX + ANU*VY)
      SIGY = EE/(1.0 - ANU**2)*(VY + ANU*UX)
      SIGXY = EE/(2.0*(1.0 + ANU))*(UY + VX)
      WRITE(6,55)I,J,M,N,EE,SIGX,SIGY,SIGXY
  200 CONTINUE
    5 FORMAT(10X,'PANEL CONNECTION POINTS ',3X,'E-MODULUS',4X,'STRESS-XX
     C',4X,'STRESS-YY',4X,'SHEAR-XY',/)
   15 FORMAT( )
   55 FORMAT(10X,4I6,4E13.4,/)
   60 FORMAT('1',/,30X,'QUA4 PLANE STRESS RECOVERY',/,30X,30('*'),/)
      RETURN
      END

      SUBROUTINE SHAPE(II,JJ,XL,YL,B,DETJAC)
C**   FIND SHAPE FUNCTIONS AND THEIR DERIVATIVES
C**   II AND JJ ARE COMMUNICATED FROM SUBROUTINE QUA4
      REAL N,NXI,NET,JAC
      DIMENSION XL(4),YL(4),B(3,8),XII(4),ETI(4)
      DIMENSION JAC(2,2),NXI(4),NET(4),N(4),AA(2)
      AA(1) = -1./SQRT(3.0)
      AA(2) = -AA(1)
      XII(1) = -1.0
      XII(2) =  1.0
      XII(3) =  1.0
      XII(4) = -1.0
      ETI(1) = -1.0
      ETI(2) = -1.0
      ETI(3) =  1.0
      ETI(4) =  1.0
      DO 10 I = 1,4
      DUM1 = (1.0 + XII(I)*AA(II))*0.25
      DUM2 = (1.0 + ETI(I)*AA(JJ))*0.25
      N(I) = 4.0*DUM1*DUM2
      NXI(I) = XII(I)*DUM2
   10 NET(I) = ETI(I)*DUM1
C**   FIND JACOBIAN,ITS INVERSE AND ITS DETERMINANT
      DO 15 I = 1,2
      DO 15 J = 1,2
   15 JAC(I,J) = 0.0
      DO 20 I = 1,4
      JAC(1,1) = JAC(1,1) + NXI(I)*XL(I)
      JAC(1,2) = JAC(1,2) + NXI(I)*YL(I)
      JAC(2,1) = JAC(2,1) + NET(I)*XL(I)
   20 JAC(2,2) = JAC(2,2) + NET(I)*YL(I)
      DETJAC = JAC(1,1)*JAC(2,2) - JAC(2,1)*JAC(1,2)
      DUM1 = JAC(1,1)/DETJAC
      JAC(1,1) = JAC(2,2)/DETJAC
      JAC(1,2) = -JAC(1,2)/DETJAC
      JAC(2,1) = -JAC(2,1)/DETJAC
      JAC(2,2) = DUM1
C**   FORM THE STRAIN-DISPLACEMENT MATRIX B.
      DO 50 L = 1,4
      J = 2*L
      I = J-1
      B(1,I) = JAC(1,1)*NXI(L) + JAC(1,2)*NET(L)
      B(1,J) = 0.0
      B(2,I) = 0.0
      B(2,J) = JAC(2,1)*NXI(L) + JAC(2,2)*NET(L)
      B(3,I) = B(2,J)
   50 B(3,J) = B(1,I)
      RETURN
      END
```

APPENDIX A2

An example illustrating the use of AIRSTR

Consider a cantilever box beam as illustrated below in Figure 1

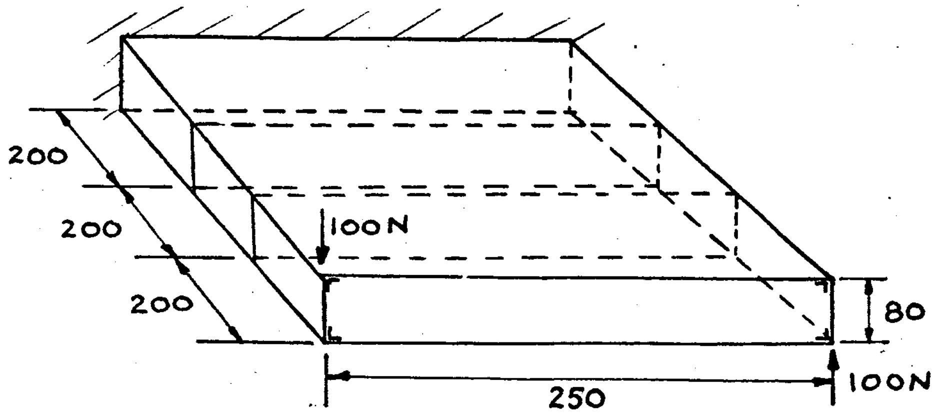
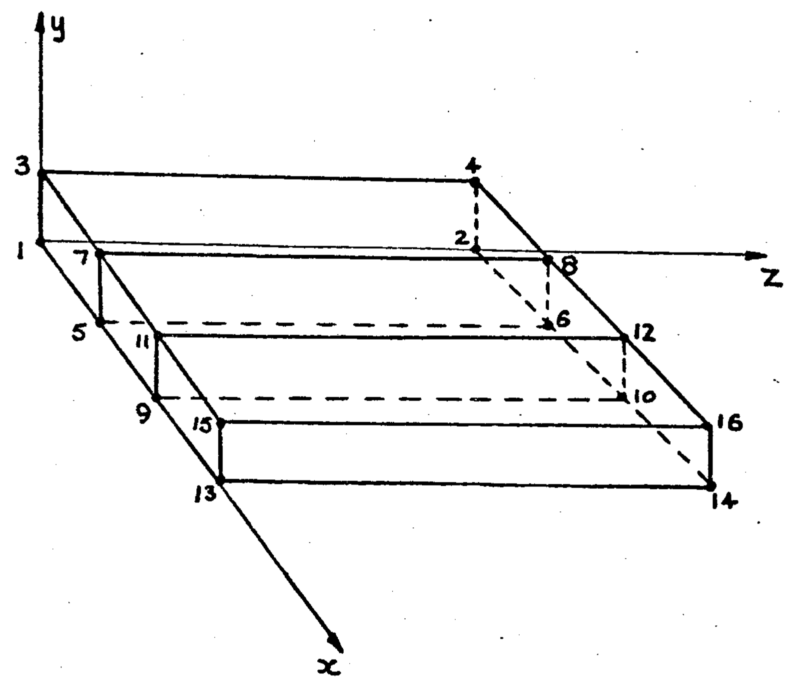


Figure 1

The beam is made up of 1 mm thick steel sheeting on the top and sides and 3 ribs which are folded over and riveted to the skin. The fold has a contact length of 10 mm with the skin. The 4 corners of the beam are stiffened by 15 x 15 x 4 mm steel angle irons.

Two loads of 100 N each are applied at the corners as shown.

Computer model



The angle stiffeners are represented by end-load elements of cross-sectional area = 104 mm^2 and an E value of 210 x 10^9 N/mm^2. They connect nodal points 1,5 ; 7,11 etc.

The rib folds are also represented by end-load elements of cross-sectional area = 10 mm^2 and they connect points 7,8 ; 5,7 ; 9,11 etc.

The rectangular panels are steel with a thickness of 1 mm,
$E = 210 \times 10^9$ N/mm^2 and
$\nu = 0{,}3$.

Three runs were made on this structure.

(a) Using end-load and 4-C.S.T. panel elements

(b) Using end-load and Idealized panel elements

(c) Using end-load and Idealized panel elements, no loads but deflections of 1 mm in the neg. y direction at nodal points 15 and 16.

A sample of the data card deck used and the complete print out from the three cases follows.

```
@RUN EXMPL,186:         ,MING,2,20
@ELT,I .ELMNTS
1  ,  5  ,  104.  ,  210.E03
5  ,  9  ,  104.  ,  210.E03
9  ,  13 ,  104.  ,  210.E03
3  ,  7  ,  104.  ,  210.E03
7  ,  11 ,  104.  ,  210.E03
11 ,  15 ,  104.  ,  210.E03
2  ,  6  ,  104.  ,  210.E03
6  ,  10 ,  104.  ,  210.E03
10 ,  14 ,  104.  ,  210.E03
4  ,  8  ,  104.  ,  210.E03
8  ,  12 ,  104.  ,  210.E03
12 ,  16 ,  104.  ,  210.E03
5  ,  7  ,  10.   ,  210.E03
9  ,  11 ,  10.   ,  210.E03
13 ,  15 ,  10.   ,  210.E03
6  ,  8  ,  10.   ,  210.E03
10 ,  12 ,  10.   ,  210.E03
14 ,  16 ,  10.   ,  210.E03
5  ,  6  ,  10.   ,  210.E03
7  ,  8  ,  10.   ,  210.E03
9  ,  10 ,  10.   ,  210.E03
11 ,  12 ,  10.   ,  210.E03
13 ,  14 ,  10.   ,  210.E03
15 ,  16 ,  10.   ,  210.E03
1  ,  5  ,  7  ,  3  ,  1.0  ,  0.3  ,  210.E03
5  ,  9  ,  11 ,  7  ,  1.0  ,  0.3  ,  210.E03
9  ,  13 ,  15 ,  11 ,  1.0  ,  0.3  ,  210.E03
2  ,  6  ,  8  ,  4  ,  1.0  ,  0.3  ,  210.E03
6  ,  10 ,  12 ,  8  ,  1.0  ,  0.3  ,  210.E03
10 ,  14 ,  16 ,  12 ,  1.0  ,  0.3  ,  210.E03
5  ,  6  ,  2  ,  1  ,  1.0  ,  0.3  ,  210.E03
9  ,  10 ,  6  ,  5  ,  1.0  ,  0.3  ,  210.E03
13 ,  14 ,  10 ,  9  ,  1.0  ,  0.3  ,  210.E03
7  ,  8  ,  4  ,  3  ,  1.0  ,  0.3  ,  210.E03
11 ,  12 ,  8  ,  7  ,  1.0  ,  0.3  ,  210.E03
15 ,  16 ,  12 ,  11 ,  1.0  ,  0.3  ,  210.E03
13 ,  14 ,  16 ,  15 ,  1.0  ,  0.3  ,  210.E03
9  ,  10 ,  12 ,  11 ,  1.0  ,  0.3  ,  210.E03
5  ,  6  ,  8  ,  7  ,  1.0  ,  0.3  ,  210.E03
@XQT  EDDY.AIRSTR
        BOX TEST BEAM (RODS + 4-CST PANELS)
16 , 24 , 0,0,15,0,0,0,0,0
1  ,  0.0  ,  0.0  ,  0.0
2  ,  0.0  ,  0.0  ,  250.
3  ,  0.0  ,  80.0 ,  0.0
4  ,  0.0  ,  80.0 ,  250.
5  ,  200. ,  0.0  ,  0.0
6  ,  200. ,  0.0  ,  250.
7  ,  200. ,  80.0 ,  0.0
8  ,  200. ,  80.0 ,  250.
9  ,  400. ,  0.0  ,  0.0
10 ,  400. ,  0.0  ,  250.
11 ,  400. ,  80.0 ,  0.0
12 ,  400. ,  80.0 ,  250.
```

```
13 , 600.  ,  0.0  ,  0.0
14 , 600.  ,  0.0  ,  250.
15 , 600.  , 80.0  ,  0.0
16 , 600.  , 80.0  ,  250.
@ADD  ELMNTS
0 , 14
1  ,  1  ,  0.0
1  ,  2  ,  0.0
1  ,  3  ,  0.0
2  ,  3  ,  0.0
2  ,  2  ,  0.0
2  ,  1  ,  0.0
3  ,  1  ,  0.0
3  ,  2  ,  0.0
3  ,  3  ,  0.0
4  ,  3  ,  0.0
4  ,  2  ,  0.0
4  ,  1  ,  0.0
16 , 2 , -1.0
15 , 2 , -1.0
@ADD  ELMNTS
@FIN
```

(v)

(a)

```
@RUN MAIN3,          ,MING,2,20

@ELT,I  ELMNTS
ELT007 R71-3A 04/06/76 18:29:46 (,0)

END ELT.   IMAGE COUNT: 39

@XQT  EDDY.AIRSTR


               *** DATA ECHC ***

    BOX TEST BEAM (RODS + 4-CST PANELS)
        16   24    0    0   15    0    0    0    0    0
          NODAL POINT COORDINATES
               1   .0000        .0000        .0000
               2   .0000        .0000        .2500+03
               3   .0000        .8000+02     .0000
               4   .0000        .8000+02     .2500+03
               5   .2000+03     .0000        .0000
               6   .2000+03     .0000        .2500+03
               7   .2000+03     .8000+02     .0000
               8   .2000+03     .8000+02     .2500+03
               9   .4000+03     .0000        .0000
              10   .4000+03     .0000        .2500+03
              11   .4000+03     .8000+02     .0000
              12   .4000+03     .8000+02     .2500+03
              13   .6000+03     .0000        .0000
              14   .6000+03     .0000        .2500+03
              15   .6000+03     .8000+02     .0000
              16   .6000+03     .8000+02     .2500+03
          FLANGE ELEMENT DATA
               1    5  .1040+03   .2100+06
               5    9  .1040+03   .2100+06
               9   13  .1040+03   .2100+06
               3    7  .1040+03   .2100+06
               7   11  .1040+03   .2100+06
              11   15  .1040+03   .2100+06
               2    6  .1040+03   .2100+06
               6   10  .1040+03   .2100+06
              10   14  .1040+03   .2100+06
               4    8  .1040+03   .2100+06
               8   12  .1040+03   .2100+06
              12   16  .1040+03   .2100+06
               5    7  .1000+02   .2100+06
               9   11  .1000+02   .2100+06
              13   15  .1000+02   .2100+06
               6    8  .1000+02   .2100+06
              10   12  .1000+02   .2100+06
              14   16  .1000+02   .2100+06
               5    6  .1000+02   .2100+06
               7    8  .1000+02   .2100+06
               9   10  .1000+02   .2100+06
              11   12  .1000+02   .2100+06
              13   14  .1000+02   .2100+06
              15   16  .1000+02   .2100+06
```

4-CST PANEL DATA

1	5	7	3	.1000+01	.3000+00	.2100+06
5	9	11	7	.1000+01	.3000+00	.2100+06
9	13	15	11	.1000+01	.3000+00	.2100+06
2	6	8	4	.1000+01	.3000+00	.2100+06
6	10	12	8	.1000+01	.3000+00	.2100+06
10	14	16	12	.1000+01	.3000+00	.2100+06
5	6	2	1	.1000+01	.3000+00	.2100+06
9	10	6	5	.1000+01	.3000+00	.2100+06
13	14	10	9	.1000+01	.3000+00	.2100+06
7	8	4	3	.1000+01	.3000+00	.2100+06
11	12	8	7	.1000+01	.3000+00	.2100+06
15	16	12	11	.1000+01	.3000+00	.2100+06
13	14	16	15	.1000+01	.3000+00	.2100+06
9	10	12	11	.1000+01	.3000+00	.2100+06
5	6	8	7	.1000+01	.3000+00	.2100+06

LOAD DATA,--NOD. PT.,DIRECTION,MAGNITUDE

14	2	.1000+03
15	2	-.1000+03

KNOWN DEFLECTIONS

1	1	.0000
1	2	.0000
1	3	.0000
2	3	.0000
2	2	.0000
2	[illegible]	.0000
3	1	.0000
3	2	.0000
3	3	.0000
4	3	.0000
4	2	.0000
4	1	.0000

(vii

BOX TEST BEAM (RODS + 4-CST PANELS)

NODAL POINT DEFLECTIONS

****

NODAL PT.	X-DEFL	Y-DEFL	Z-DEFL
1	.0000	.0000	.0000
2	.0000	.0000	.0000
3	.0000	.0000	.0000
4	.0000	.0000	.0000
5	-.2662-03	-.2637-02	.9106-03
6	.2663-03	.2637-02	.9112-03
7	.2663-03	-.2637-02	-.9112-03
8	-.2662-03	.2637-02	-.9106-03
9	-.3618-03	-.5847-02	.1870-02
10	.3614-03	.5839-02	.1865-02
11	.3614-03	-.5839-02	-.1865-02
12	-.3618-03	.5847-02	-.1870-02
13	-.3914-03	-.9336-02	.2699-02
14	.3977-03	.9423-02	.2724-02
15	.3977-03	-.9423-02	-.2724-02
16	-.3914-03	.9336-02	-.2699-02

BOX TEST BEAM (RODS + 4-CST PANELS)

SEMI-MONOCOQUE STRUCTURAL ANALYSIS
****

ROD	NOD.PTS	AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
1	5	.1040+03	.2100+06	.2000+03	-.2907+02	-.2796+00
5	9	.1040+03	.2100+06	.2000+03	-.1044+02	-.1004+00
9	13	.1040+03	.2100+06	.2000+03	-.3231+01	-.3107-01
3	7	.1040+03	.2100+06	.2000+03	.2908+02	.2796+00
7	11	.1040+03	.2100+06	.2000+03	.1039+02	.9991-01
11	15	.1040+03	.2100+06	.2000+03	.3959+01	.3806-01
2	6	.1040+03	.2100+06	.2000+03	.2908+02	.2796+00
6	10	.1040+03	.2100+06	.2000+03	.1039+02	.9991-01
10	14	.1040+03	.2100+06	.2000+03	.3959+01	.3806-01
4	8	.1040+03	.2100+06	.2000+03	-.2907+02	-.2796+00
8	12	.1040+03	.2100+06	.2000+03	-.1044+02	-.1004+00
12	16	.1040+03	.2100+06	.2000+03	-.3231+01	-.3107-01
5	7	.1000+02	.2100+06	.8000+02	-.2172-01	-.2172-02
9	11	.1000+02	.2100+06	.8000+02	.2214+00	.2214-01
13	15	.1000+02	.2100+06	.8000+02	-.2268+01	-.2268+00
6	8	.1000+02	.2100+06	.8000+02	-.2172-01	-.2172-02
10	12	.1000+02	.2100+06	.8000+02	.2214+00	.2214-01
14	16	.1000+02	.2100+06	.8000+02	-.2268+01	-.2268+00
5	6	.1000+02	.2100+06	.2500+03	.5151-02	.5151-03
7	8	.1000+02	.2100+06	.2500+03	.5158-02	.5158-03
9	10	.1000+02	.2100+06	.2500+03	-.3777-01	-.3777-02
11	12	.1000+02	.2100+06	.2500+03	-.3777-01	-.3777-02
13	14	.1000+02	.2100+06	.2500+03	.2084+00	.2084-01
15	16	.1000+02	.2100+06	.2500+03	.2084+00	.2084-01

4-CST PANEL STRESS RECOVERY
****

ELEMENT CONN.PTS 1 5 7 3

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	-.1331-05	.8143-06	-.9857-05	-.2508+00	.9575-01	-.7962+00
2	.1118-08	-.1034-07	-.6856-05	-.4582-03	-.2310-02	-.5538+00
3	.1331-05	-.8246-06	-.9856-05	.2501+00	-.9813-01	-.7961+00
4	-.9919-09	.1962-12	-.1286-04	-.2289-03	-.6863-04	-.1038+01

AVSIGX= -.3436-03 AVSIGY= -.1189-02 AVSIGXY= -.7961+00

ELEMENT CONN.PTS 5 9 11 7

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	-.4779-06	.3410-06	-.8174-05	-.8668-01	.4562-01	-.6602+00
2	-.1288-07	.1054-06	-.7106-05	.4327-02	.2344-01	-.5739+00
3	.4758-06	-.2460-06	-.8187-05	.9276-01	-.2383-01	-.6612+00
4	.1073-07	-.1034-07	-.9255-05	.1760-02	-.1644-02	-.7475+00

AVSIGX= .3043-02 AVSIGY= .1090-01 AVSIGXY= -.6607+00

ELEMENT CONN.PTS 9 13 15 11

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	-.1480-06	-.3859-06	-.8296-05	-.6086-01	-.9929-01	-.6701+00
2	.1375-06	-.1080-05	-.7860-05	-.4303-01	-.2397+00	-.6349+00
3	.1813-06	-.5885-06	-.8166-05	.1083-02	-.1233+00	-.6596+00
4	-.1042-06	.1054-06	-.8602-05	-.1675-01	.1711-01	-.6948+00

AVSIGX= -.2989-01 AVSIGY= -.1113+00 AVSIGXY= -.6648+00

ELEMENT CONN.PTS 2 6 8 4

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY

1	.1331-05	-.3246-06	.9856-05	.2501+00	-.9813-01	.7961+00
2	.1115-03	-.1034-07	.6856-05	-.4587-03	-.2310-02	.5538+00
3	-.1331-05	.8143-06	.9857-05	-.2508+00	.9575-01	.7962+00
4	-.9939-09	-.1962-12	.1286-04	-.2294-03	-.6885-04	.1038+01

AVSIGX= -.3441-03 AVSIGY= -.1189-02 AVSIGXY= .7961+00

ELEMENT CONN.PTS 6 10 12 8

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	.4758-06	-.2460-06	.8187-05	.9276-01	-.2383-01	.6612+00
2	-.1288-07	.1054-06	.7106-05	.4326-02	.2344-01	.5739+00
3	-.4779-06	.3411-06	.8174-05	-.8668-01	.4562-01	.6602+00
4	.1073-07	-.1034-07	.9255-05	.1759-02	-.1644-02	.7475+00

AVSIGX= .3043-02 AVSIGY= .1090-01 AVSIGXY= .6607+00

ELEMENT CONN.PTS 10 14 16 12

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	.1813-06	-.5865-06	.8166-05	.1083-02	-.1233+00	.6596+00
2	.1375-06	-.1080-05	.7860-05	-.4303-01	-.2397+00	.6349+00
3	-.1480-06	-.3859-06	.8297-05	-.6086-01	-.9929-01	.6701+00
4	-.1042-06	.1054-06	.8602-05	-.1675-01	.1712-01	.6948+00

AVSIGX= -.2989-01 AVSIGY= -.1113+00 AVSIGXY= .6648+00

ELEMENT CONN.PTS 5 6 2 1

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	.2453-08	-.5814-09	-.5985-05	.5258-03	.3565-04	-.4834+00
2	-.5582-06	.1331-05	-.5620-05	-.3664-01	.2686+00	-.4540+00
3	.0000	.7212-09	-.5254-05	.4993-04	.1664-03	-.4243+00
4	.5606-06	-.1331-05	-.5618-05	.3722-01	-.2684+00	-.4538+00

AVSIGX= .2879-03 AVSIGY= .1010-03 AVSIGXY= -.4539+00

ELEMENT CONN.PTS 9 10 6 5

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	-.1798-07	.4355-08	-.7426-05	-.3849-02	-.2401-03	-.5998+00
2	-.2081-06	.4758-06	-.7287-05	-.1509-01	.9538-01	-.5886+00
3	.2456-08	-.6498-08	-.7164-05	.1162-03	-.1330-02	-.5787+00
4	.1926-06	-.4779-06	-.7304-05	.1136-01	-.9695-01	-.5899+00

AVSIGX= -.1866-02 AVSIGY= -.7849-03 AVSIGXY= -.5892+00

ELEMENT CONN.PTS 13 14 10 9

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	.9925-07	-.1448-07	-.7290-05	.2190-01	.3530-02	-.5888+00
2	-.2853-07	.1813-06	-.7293-05	.5964-02	.3985-01	-.5891+00
3	-.1798-07	.4778-07	-.7199-05	-.8426-03	.9781-02	-.5815+00
4	.1098-06	-.1480-06	-.7196-05	.1510-01	-.2654-01	-.5812+00

AVSIGX= .1053-01 AVSIGY= .6655-02 AVSIGXY= -.5851+00

ELEMENT CONN.PTS 7 8 4 3

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	.2456-08	-.5985-09	.5985-05	.5254-03	.3192-04	.4834+00
2	.5606-06	-.1331-05	.5618-05	.3722-01	-.2684+00	.4538+00
3	.0000	.7058-09	.5254-05	.4886-04	.1629-03	.4243+00
4	-.5582-06	.1331-05	.5620-05	-.3664-01	.2686+00	.4540+00

AVSIGX= .2871-03 AVSIGY= .9740-04 AVSIGXY= .4539+00

ELEMENT CONN.PTS 11 12 8 7

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	-.1798-07	.4347-08	.7426-05	-.3849-02	-.2418-03	.5998+00
2	.1926-06	-.4779-06	.7304-05	.1136-01	-.9696-01	.5899+00
3	.2456-08	-.6507-08	.7164-05	.1163-03	-.1332-02	.5787+00
4	-.2081-06	.4758-06	.7287-05	-.1509-01	.9538-01	.5886+00

AVSIGX= -.1866-02 AVSIGY= -.7867-03 AVSIGXY= .5892+00

ELEMENT CONN.PTS 15 16 12 11

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	.9925-07	-.1448-07	.7290-05	.2190-01	.3530-02	.5888+00
2	.1098-06	-.1480-06	.7196-05	.1510-01	-.2654-01	.5812+00
3	-.1798-07	.4778-07	.7199-05	-.8425-03	.9780-02	.5815+00
4	-.2853-07	.1813-06	.7293-05	.5964-02	.3985-01	.5891+00

AVSIGX= .1053-01 AVSIGY= .6655-02 AVSIGXY= .5851+00

ELEMENT CONN.PTS 13 14 16 15

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	.9925-07	-.1080-05	.7245-05	-.5185-01	-.2423+00	.5851+00
2	.9925-07	-.1080-05	.7245-05	-.5185-01	-.2423+00	.5851+00
3	.9925-07	-.1080-05	.7245-05	-.5185-01	-.2423+00	.5851+00
4	.9925-07	-.1080-05	.7245-05	-.5185-01	-.2423+00	.5851+00

AVSIGX= -.5185-01 AVSIGY= -.2423+00 AVSIGXY= .5851+00

ELEMENT CONN.PTS 9 10 12 11

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	-.1798-07	.1054-06	.5081-07	.3148-02	.2308-01	.4104-02
2	-.1798-07	.1054-06	.5081-07	.3148-02	.2308-01	.4104-02
3	-.1798-07	.1054-06	.5080-07	.3149-02	.2308-01	.4103-02
4	-.1798-07	.1054-06	.5080-07	.3148-02	.2308-01	.4103-02

AVSIGX= .3148-02 AVSIGY= .2308-01 AVSIGXY= .4103-02

ELEMENT CONN.PTS 5 6 3 7

TRIANGLE	EPSX	EPSY	EPSXY	SIGX	SIGY	SIGXY
1	.2453-08	-.1034-07	-.1676-05	-.1500-03	-.2217-02	-.1354+00
2	.2454-08	-.1034-07	-.1676-05	-.1497-03	-.2217-02	-.1354+00

3	.2456-08	-.1034-07	-.1676-05	-.1494-03	-.2217-02	-.1354+00
4	.2455-08	-.1034-07	-.1676-05	-.1497-03	-.2217-02	-.1354+00

AVSIGX= -.1497-03 AVSIGY= -.2217-02 AVSIGXY= -.1354+00

@FIN

```
RUNID: MAIN3     ACCT: 1861002        PROJECT: 186-MING
TIME:    TOTAL: 00:00:06.141      CBSUPS:  004516713
         CAU:   00:00:02.322      I/O:  00:00:01.849
         CC/ER: 00:00:01.969      WAIT: 00:00:00.000

SUPS USED:          0.06   SUPS REMAINING:       191.06
SRC:     PS=       000000000      ES=        005306489
IMAGES  READ:   156        PAGES:   9
START:     18:29:45 APR 06,1976    FIN:  18:30:04 APR 06,1976
```

(b)

```
@RUN MAIN3,          ,MING,2,20

@ELT,I  ELMNTS
ELT007 R71-3A 04/06/76 18:37:38 (,0)

END ELT.   IMAGE COUNT: 39

@XQT  EDDY,AIRSTR
```

*** DATA ECHO ***

BOX TEST BEAM (RODS + IDEALIZED QUAD. PANELS)

16	24	0	0	0	0	0	15	0	0

NODAL POINT COORDINATES

1	.0000	.0000	.0000
2	.0000	.0000	.2500+03
3	.0000	.8000+02	.0000
4	.0000	.8000+02	.2500+03
5	.2000+03	.0000	.0000
6	.2000+03	.0000	.2500+03
7	.2000+03	.8000+02	.0000
8	.2000+03	.8000+02	.2500+03
9	.4000+03	.0000	.0000
10	.4000+03	.0000	.2500+03
11	.4000+03	.8000+02	.0000
12	.4000+03	.8000+02	.2500+03
13	.6000+03	.0000	.0000
14	.6000+03	.0000	.2500+03
15	.6000+03	.8000+02	.0000
16	.6000+03	.8000+02	.2500+03

FLANGE ELEMENT DATA

1	5	.1040+03	.2100+06
5	9	.1040+03	.2100+06
9	13	.1040+03	.2100+06
3	7	.1040+03	.2100+06
7	11	.1040+03	.2100+06
11	15	.1040+03	.2100+06
2	6	.1040+03	.2100+06
6	10	.1040+03	.2100+06
10	14	.1040+03	.2100+06
4	8	.1040+03	.2100+06
8	12	.1040+03	.2100+06
12	16	.1040+03	.2100+06
5	7	.1000+02	.2100+06
9	11	.1000+02	.2100+06
13	15	.1000+02	.2100+06
6	8	.1000+02	.2100+06
10	12	.1000+02	.2100+06
14	16	.1000+02	.2100+06
5	6	.1000+02	.2100+06
7	8	.1000+02	.2100+06
9	10	.1000+02	.2100+06
11	12	.1000+02	.2100+06
13	14	.1000+02	.2100+06
15	16	.1000+02	.2100+06

IDEALIZED PANEL DATA

1	5	7	3	.1000+01	.3000+00	.2100+06
5	9	11	7	.1000+01	.3000+00	.2100+06
9	13	15	11	.1000+01	.3000+00	.2100+06
2	6	8	4	.1000+01	.3000+00	.2100+06
6	10	12	8	.1000+01	.3000+00	.2100+06
10	14	16	12	.1000+01	.3000+00	.2100+06
5	6	2	1	.1000+01	.3000+00	.2100+06
9	10	6	5	.1000+01	.3000+00	.2100+06
13	14	10	9	.1000+01	.3000+00	.2100+06
7	8	4	3	.1000+01	.3000+00	.2100+06
11	12	8	7	.1000+01	.3000+00	.2100+06
15	16	12	11	.1000+01	.3000+00	.2100+06
13	14	16	15	.1000+01	.3000+00	.2100+06
9	10	12	11	.1000+01	.3000+00	.2100+06
5	6	8	7	.1000+01	.3000+00	.2100+06

LOAD DATA,--NOD. PT.,DIRECTION,MAGNITUDE

14	2	.1000+03
15	2	-.1000+03

KNOWN DEFLECTIONS

1	1	.0000
1	2	.0000
1	3	.0000
2	3	.0000
2	2	.0000
2	1	.0000
3	1	.0000
3	2	.0000
3	3	.0000
4	3	.0000
4	2	.0000
4	1	.0000

BOX TEST BEAM (RODS + IDEALIZED QUAD. PANELS)
NODAL POINT DEFLECTIONS
****

NODAL PT.	X-DEFL	Y-DEFL	Z-DEFL
1	.0000	.0000	.0000
2	.0000	.0000	.0000
3	.0000	.0000	.0000
4	.0000	.0000	.0000
5	-.2928-03	-.2669-02	.9237-03
6	.2928-03	.2669-02	.9237-03
7	.2928-03	-.2669-02	-.9237-03
8	-.2928-03	.2669-02	-.9237-03
9	-.3768-03	-.5933-02	.1894-02
10	.3768-03	.5933-02	.1894-02
11	.3768-03	-.5933-02	-.1894-02
12	-.3768-03	.5933-02	-.1894-02
13	-.4087-03	-.9398-02	.2747-02
14	.4087-03	.9622-02	.2747-02
15	.4087-03	-.9622-02	-.2747-02
16	-.4087-03	.9398-02	-.2747-02

BOX TEST BEAM (RODS + IDEALIZED QUAD. PANELS)

SEMI-MONOCOQUE STRUCTURAL ANALYSIS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
1	5	.1040+03	.2100+06	.2000+03	-.3197+02	-.3074+00
5	9	.1040+03	.2100+06	.2000+03	-.9177+01	-.8824-01
9	13	.1040+03	.2100+06	.2000+03	-.3482+01	-.3348-01
3	7	.1040+03	.2100+06	.2000+03	.3197+02	.3074+00
7	11	.1040+03	.2100+06	.2000+03	.9177+01	.8824-01
11	15	.1040+03	.2100+06	.2000+03	.3482+01	.3348-01
2	6	.1040+03	.2100+06	.2000+03	.3197+02	.3074+00
6	10	.1040+03	.2100+06	.2000+03	.9177+01	.8824-01
10	14	.1040+03	.2100+06	.2000+03	.3482+01	.3348-01
4	8	.1040+03	.2100+06	.2000+03	-.3197+02	-.3074+00
8	12	.1040+03	.2100+06	.2000+03	-.9177+01	-.8824-01
12	16	.1040+03	.2100+06	.2000+03	-.3482+01	-.3348-01
5	7	.1000+02	.2100+06	.8000+02	.0000	.0000
9	11	.1000+02	.2100+06	.8000+02	-.6112-05	-.6112-06
13	15	.1000+02	.2100+06	.8000+02	-.5882+01	-.5882+00
6	8	.1000+02	.2100+06	.8000+02	.1528-05	.1528-06
10	12	.1000+02	.2100+06	.8000+02	.3056-05	.3056-06
14	16	.1000+02	.2100+06	.8000+02	-.5882+01	-.5882+00
5	6	.1000+02	.2100+06	.2500+03	-.3056-06	-.3056-07
7	8	.1000+02	.2100+06	.2500+03	.1834-06	.1834-07
9	10	.1000+02	.2100+06	.2500+03	.0000	.0000
11	12	.1000+02	.2100+06	.2500+03	.0000	.0000
13	14	.1000+02	.2100+06	.2500+03	-.4889-06	-.4889-07
15	16	.1000+02	.2100+06	.2500+03	.0000	.0000

SHEAR PANEL NOD.PTS				B-LENGTH	HEIGHT	G-VALUE	THICKNESS	EPSXY	SHEAR STRESS
1	5	7	3	.2000+03	.8000+02	.8077+05	.1000+01	-.9686-05	-.7823+00

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
1	5	.1333+02	.2100+06	.2000+03	-.4099+01	-.3074+00
3	7	.1333+02	.2100+06	.2000+03	.4099+01	.3074+00
5	7	.3333+02	.2100+06	.8000+02	.0000	.0000
1	3	.3333+02	.2100+06	.8000+02	.0000	.0000

5	9	11	7	.2000+03	.8000+02	.8077+05	.1000+01	-.7947-05	-.6419+00

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
5	9	.1333+02	.2100+06	.2000+03	-.1177+01	-.8824-01
7	11	.1333+02	.2100+06	.2000+03	.1177+01	.8824-01
9	11	.3333+02	.2100+06	.8000+02	-.2037-04	-.6112-06
5	7	.3333+02	.2100+06	.8000+02	.0000	.0000

9	13	15	11	.2000+03	.8000+02	.8077+05	.1000+01	-.8068-05	-.6516+00

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
9	13	.1333+02	.2100+06	.2000+03	-.4464+00	-.3348-01
11	15	.1333+02	.2100+06	.2000+03	.4464+00	.3348-01
13	15	.3333+02	.2100+06	.8000+02	-.1961+02	-.5882+00
9	11	.3333+02	.2100+06	.8000+02	-.2037-04	-.6112-06

2	6	8	4	.2000+03	.8000+02	.8077+05	.1000+01	.9686-05	.7823+00

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS

2	6	.1333+02	.2100+06	.2000+03	.4099+01	.3074+00
4	8	.1333+02	.2100+06	.2000+03	-.4099+01	-.3074+00
6	8	.3333+02	.2100+06	.8000+02	.5093-05	.1528-06
2	4	.3333+02	.2100+06	.8000+02	.0000	.0000

6	10	12	8	.2000+03	.8000+02	.8077+05	.1000+01	.7947-05	.6419+00

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
6	10	.1333+02	.2100+06	.2000+03	.1177+01	.8824-01
8	12	.1333+02	.2100+06	.2000+03	-.1177+01	-.8824-01
10	12	.3333+02	.2100+06	.8000+02	.1019-04	.3056-06
6	8	.3333+02	.2100+06	.8000+02	.5093-05	.1528-06

10	14	16	12	.2000+03	.8000+02	.3077+05	.1000+01	.8068-05	.6516+00

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
10	14	.1333+02	.2100+06	.2000+03	.4464+00	.3348-01
12	16	.1333+02	.2100+06	.2000+03	-.4464+00	-.3348-01
14	16	.3333+02	.2100+06	.8000+02	-.1961+02	-.5882+00
10	12	.3333+02	.2100+06	.8000+02	.1019-04	.3056-06

5	6	2	1	.2500+03	.2000+03	.8077+05	.1000+01	-.5790-05	-.4676+00

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
5	6	.3333+02	.2100+06	.2500+03	-.1019-05	-.3056-07
1	2	.3333+02	.2100+06	.2500+03	.0000	.0000
6	2	.4167+02	.2100+06	.2000+03	.1281+02	.3074+00
5	1	.4167+02	.2100+06	.2000+03	-.1281+02	-.3074+00

```
   9   10    6    5   .2500+03   .2000+03   .8077+05   .1000+01   -.7529-05   -.6081+00
                  IDEALIZED PANEL STIFFENER LOADS
```

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
9	10	.3333+02	.2100+06	.2500+03	.0000	.0000
5	6	.3333+02	.2100+06	.2500+03	-.1019-05	-.3056-07
10	6	.4167+02	.2100+06	.2000+03	.3677+01	.8824-01
9	5	.4167+02	.2100+06	.2000+03	-.3677+01	-.8824-01

```
  13   14   10    9   .2500+03   .2000+03   .8077+05   .1000+01   -.7409-05   -.5984+00
                  IDEALIZED PANEL STIFFENER LOADS
```

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
13	14	.3333+02	.2100+06	.2500+03	-.1630-05	-.4889-07
9	10	.3333+02	.2100+06	.2500+03	.0000	.0000
14	10	.4167+02	.2100+06	.2000+03	.1395+01	.3348-01
13	9	.4167+02	.2100+06	.2000+03	-.1395+01	-.3348-01

```
   7    8    4    3   .2500+03   .2000+03   .8077+05   .1000+01    .5790-05    .4676+00
                  IDEALIZED PANEL STIFFENER LOADS
```

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
7	8	.3333+02	.2100+06	.2500+03	.6112-06	.1834-07
3	4	.3333+02	.2100+06	.2500+03	.0000	.0000
8	4	.4167+02	.2100+06	.2000+03	-.1281+02	-.3074+00
7	3	.4167+02	.2100+06	.2000+03	.1281+02	.3074+00

```
  11   12    8    7   .2500+03   .2000+03   .8077+05   .1000+01    .7529-05    .6081+00
                  IDEALIZED PANEL STIFFENER LOADS
```

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
11	12	.3333+02	.2100+06	.2500+03	.0000	.0000

7	8	.3333+02	.2100+06	.2500+03	.6112-06	.1834-07
12	8	.4167+02	.2100+06	.2000+03	-.3677+01	-.8824-01
11	7	.4167+02	.2100+06	.2000+03	.3677+01	.8824-01

15 16 12 11 .2500+03 .2000+03 .8077+05 .1000+01 .7409-05 .5984+00

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
15	16	.3333+02	.2100+06	.2500+03	.0000	.0000
11	12	.3333+02	.2100+06	.2500+03	.0000	.0000
16	12	.4167+02	.2100+06	.2000+03	-.1395+01	-.3348-01
15	11	.4167+02	.2100+06	.2000+03	.1395+01	.3348-01

13 14 16 15 .2500+03 .8000+02 .8077+05 .1000+01 .7409-05 .5984+00

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
13	14	.1333+02	.2100+06	.2500+03	-.6519-06	-.4889-07
15	16	.1333+02	.2100+06	.2500+03	.0000	.0000
14	16	.4167+02	.2100+06	.8000+02	-.2451+02	-.5882+00
13	15	.4167+02	.2100+06	.8000+02	-.2451+02	-.5882+00

9 10 12 11 .2500+03 .8000+02 .8077+05 .1000+01 .1201-06 .9699-02

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
9	10	.1333+02	.2100+06	.2500+03	.0000	.0000
11	12	.1333+02	.2100+06	.2500+03	.0000	.0000
10	12	.4167+02	.2100+06	.8000+02	.1273-04	.3056-06
9	11	.4167+02	.2100+06	.8000+02	-.2547-04	-.6112-06

```
5    6    8    7    .2500+03    .8000+02    .8077+05    .1000+01    -.1739-05    -.1404+00
```

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
5	6	.1333+02	.2100+06	.2500+03	-.4075-06	-.3056-07
7	8	.1333+02	.2100+06	.2500+03	.2445-06	.1834-07
6	8	.4167+02	.2100+06	.8000+02	.6366-05	.1528-06
5	7	.4167+02	.2100+06	.8000+02	.0000	.0000

```
@FIN

RUNID: MAIN3    ACCT: 1861002         PROJECT: 186-MING
TIME:     TOTAL: 00:00:05.848     CBSUPS:  004277773
          CAU:   00:00:02.014     I/O:   00:00:01.849
          CC/ER: 00:00:01.985     WAIT:  00:00:00.000

SUPS USED:          0.05   SUPS REMAINING:       191.01
SRC:      PS=      000000000      ES=        004602085
IMAGES  READ:    156        PAGES:   9
START:     18:37:37 APR 06,1976     FIN:   16:37:55 APR 06,1976
```

```
@RUN MAIN3,          ,MING,2,20

@ELT,I  ELMNTS
ELT007 R71-3A 04/13/76 17:05:43 (,0)

END ELT.   IMAGE COUNT: 39

@XQT  EDDY.AIRSTR


                    *** DATA ECHO ***

   BOX TEST BEAM (RODS + IDEALIZED QUAD. PANELS)
            16    24     0     0     0     0     0    15     0     0
        NODAL POINT COORDINATES

             1      .0000         .0000         .0000
             2      .0000         .0000         .2500+03
             3      .0000         .8000+02      .0000
             4      .0000         .8000+02      .2500+03
             5      .2000+03      .0000         .0000
             6      .2000+03      .0000         .2500+03
             7      .2000+03      .8000+02      .0000
             8      .2000+03      .8000+02      .2500+03
             9      .4000+03      .0000         .0000
            10      .4000+03      .0000         .2500+03
            11      .4000+03      .8000+02      .0000
            12      .4000+03      .8000+02      .2500+03
            13      .6000+03      .0000         .0000
            14      .6000+03      .0000         .2500+03
            15      .6000+03      .8000+02      .0000
            16      .6000+03      .8000+02      .2500+03
        FLANGE ELEMENT DATA

             1     5    .1040+03     .2100+06
             5     9    .1040+03     .2100+06
             9    13    .1040+03     .2100+06
             3     7    .1040+03     .2100+06
             7    11    .1040+03     .2100+06
            11    15    .1040+03     .2100+06
             2     6    .1040+03     .2100+06
             6    10    .1040+03     .2100+06
            10    14    .1040+03     .2100+06
             4     8    .1040+03     .2100+06
             8    12    .1040+03     .2100+06
            12    16    .1040+03     .2100+06
             5     7    .1000+02     .2100+06
             9    11    .1000+02     .2100+06
            13    15    .1000+02     .2100+06
             6     8    .1000+02     .2100+06
            10    12    .1000+02     .2100+06
            14    16    .1000+02     .2100+06
             5     6    .1000+02     .2100+06
             7     8    .1000+02     .2100+06
             9    10    .1000+02     .2100+06
            11    12    .1000+02     .2100+06
            13    14    .1000+02     .2100+06
            15    16    .1000+02     .2100+06
```

IDEALIZED PANEL DATA

1	5	7	3	.1000+01	.3000+00	.2100+06
5	9	11	7	.1000+01	.3000+00	.2100+06
9	13	15	11	.1000+01	.3000+00	.2100+06
2	6	8	4	.1000+01	.3000+00	.2100+06
6	10	12	8	.1000+01	.3000+00	.2100+06
10	14	16	12	.1000+01	.3000+00	.2100+06
5	6	2	1	.1000+01	.3000+00	.2100+06
9	10	6	5	.1000+01	.3000+00	.2100+06
13	14	10	9	.1000+01	.3000+00	.2100+06
7	8	4	3	.1000+01	.3000+00	.2100+06
11	12	8	7	.1000+01	.3000+00	.2100+06
15	16	12	11	.1000+01	.3000+00	.2100+06
13	14	16	15	.1000+01	.3000+00	.2100+06
9	10	12	11	.1000+01	.3000+00	.2100+06
5	6	8	7	.1000+01	.3000+00	.2100+06

KNOWN DEFLECTIONS

1	1	.0000
1	2	.0000
1	3	.0000
2	3	.0000
2	2	.0000
2	1	.0000
3	1	.0000
3	2	.0000
3	3	.0000
4	3	.0000
4	2	.0000
4	1	.0000
16	2	-.1000+01
15	2	-.1000+01

BOX TEST BEAM (RODS + IDEALIZED QUAD. PANELS)

NODAL POINT DEFLECTIONS

****

NODAL PT.	X-DEFL	Y-DEFL	Z-DEFL
1	.0000	.0000	.0000
2	.0000	.0000	.0000
3	.0000	.0000	.0000
4	.0000	.0000	.0000
5	-.4997-01	-.1663+00	.5786-07
6	-.4997-01	-.1663+00	.5828-07
7	.4997-01	-.1663+00	.2810-07
8	.4997-01	-.1663+00	.2835-07
9	-.7996-01	-.5324+00	.1335-06
10	-.7996-01	-.5324+00	.1315-06
11	.7996-01	-.5324+00	.9053-07
12	.7996-01	-.5324+00	.9031-07
13	-.8995-01	-.9970+00	.1801-06
14	-.8995-01	-.9970+00	.1763-06
15	.8995-01	-.1000+01	.1713-06
16	.8995-01	-.1000+01	.1716-06

BOX TEST BEAM (RODS + IDEALIZED QUAD. PANELS)

SEMI-MONOCOQUE STRUCTURAL ANALYSIS
****

ROD NOD.PTS	AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
1 5	.1040+03	.2100+06	.2000+03	-.5457+04	-.5247+02
5 9	.1040+03	.2100+06	.2000+03	-.3274+04	-.3148+02
9 13	.1040+03	.2100+06	.2000+03	-.1091+04	-.1049+02
3 7	.1040+03	.2100+06	.2000+03	.5457+04	.5247+02
7 11	.1040+03	.2100+06	.2000+03	.3274+04	.3148+02
11 15	.1040+03	.2100+06	.2000+03	.1091+04	.1049+02
2 6	.1040+03	.2100+06	.2000+03	-.5457+04	-.5247+02
6 10	.1040+03	.2100+06	.2000+03	-.3274+04	-.3148+02
10 14	.1040+03	.2100+06	.2000+03	-.1091+04	-.1049+02
4 8	.1040+03	.2100+06	.2000+03	.5457+04	.5247+02
8 12	.1040+03	.2100+06	.2000+03	.3274+04	.3148+02
12 16	.1040+03	.2100+06	.2000+03	.1091+04	.1049+02
5 7	.1000+02	.2100+06	.8000+02	.4889-04	.4889-05
9 11	.1000+02	.2100+06	.8000+02	.0000	.0000
13 15	.1000+02	.2100+06	.8000+02	-.7853+02	-.7853+01
6 8	.1000+02	.2100+06	.8000+02	.4889-04	.4889-05
10 12	.1000+02	.2100+06	.8000+02	.0000	.0000
14 16	.1000+02	.2100+06	.8000+02	-.7853+02	-.7853+01
5 6	.1000+02	.2100+06	.2500+03	.3545-05	.3545-06
7 8	.1000+02	.2100+06	.2500+03	.2105-05	.2105-06
9 10	.1000+02	.2100+06	.2500+03	-.1644-04	-.1644-05
11 12	.1000+02	.2100+06	.2500+03	-.1865-05	-.1865-06
13 14	.1000+02	.2100+06	.2500+03	-.3176-04	-.3176-05
15 16	.1000+02	.2100+06	.2500+03	.2931-05	.2931-06

SHEAR PANEL NOD.PTS				B-LENGTH	HEIGHT	G-VALUE	THICKNESS	EPSXY	SHEAR STRESS
1	5	7	3	.2000+03	.8000+02	.8077+05	.1000+01	-.2066-03	-.1669+02

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
1	5	.1333+02	.2100+06	.2000+03	-.6996+03	-.5247+02
3	7	.1333+02	.2100+06	.2000+03	.6996+03	.5247+02
5	7	.3333+02	.2100+06	.8000+02	.1630-03	.4889-05
1	3	.3333+02	.2100+06	.8000+02	.0000	.0000

5	9	11	7	.2000+03	.8000+02	.8077+05	.1000+01	-.2066-03	-.1669+02

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
5	9	.1333+02	.2100+06	.2000+03	-.4198+03	-.3148+02
7	11	.1333+02	.2100+06	.2000+03	.4198+03	.3148+02
9	11	.3333+02	.2100+06	.8000+02	.0000	.0000
5	7	.3333+02	.2100+06	.8000+02	.1630-03	.4889-05

9	13	15	11	.2000+03	.8000+02	.8077+05	.1000+01	-.2066-03	-.1669+02

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
9	13	.1333+02	.2100+06	.2000+03	-.1399+03	-.1049+02
11	15	.1333+02	.2100+06	.2000+03	.1399+03	.1049+02
13	15	.3333+02	.2100+06	.8000+02	-.2618+03	-.7853+01
9	11	.3333+02	.2100+06	.8000+02	.0000	.0000

2	6	8	4	.2000+03	.8000+02	.8077+05	.1000+01	-.2066-03	-.1669+02

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS	AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS

2	6	.1333+02	.2100+06	.2000+03	-.6996+03	-.5247+02
4	8	.1333+02	.2100+06	.2000+03	.6996+03	.5247+02
6	8	.3333+02	.2100+06	.8000+02	.1630-03	.4889-05
2	4	.3333+02	.2100+06	.8000+02	.0000	.0000

6 10 12 8 .2000+03 .8000+02 .8077+05 .1000+01 -.2066-03 -.1669+02

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
6	10	.1333+02	.2100+06	.2000+03	-.4198+03	-.3148+02
8	12	.1333+02	.2100+06	.2000+03	.4198+03	.3148+02
10	12	.3333+02	.2100+06	.8000+02	.0000	.0000
6	8	.3333+02	.2100+06	.8000+02	.1630-03	.4889-05

10 14 16 12 .2000+03 .8000+02 .8077+05 .1000+01 -.2066-03 -.1669+02

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
10	14	.1333+02	.2100+06	.2000+03	-.1399+03	-.1049+02
12	16	.1333+02	.2100+06	.2000+03	.1399+03	.1049+02
14	16	.3333+02	.2100+06	.8000+02	-.2618+03	-.7853+01
10	12	.3333+02	.2100+06	.8000+02	.0000	.0000

5 6 2 1 .2500+03 .2000+03 .8077+05 .1000+01 -.2194-09 -.1772-04

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
5	6	.3333+02	.2100+06	.2500+03	.1182-04	.3545-06
1	2	.3333+02	.2100+06	.2500+03	.0000	.0000
6	2	.4167+02	.2100+06	.2000+03	-.2186+04	-.5247+02
5	1	.4167+02	.2100+06	.2000+03	-.2186+04	-.5247+02

9	10	6	5	.2500+03	.2000+03	.8077+05	.1000+01	-.1557-09	-.1258-04

IDEALIZED PANEL STIFFENER LOADS

ROD	NOD.PTS	AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
9	10	.3333+02	.2100+06	.2500+03	-.5480-04	-.1644-05
5	6	.3333+02	.2100+06	.2500+03	.1182-04	.3545-05
10	6	.4167+02	.2100+06	.2000+03	-.1312+04	-.3148+02
9	5	.4167+02	.2100+06	.2000+03	-.1312+04	-.3148+02

13	14	10	9	.2500+03	.2000+03	.8077+05	.1000+01	.1013-09	.8181-05

IDEALIZED PANEL STIFFENER LOADS

ROD	NOD.PTS	AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
13	14	.3333+02	.2100+06	.2500+03	-.1059-03	-.3176-05
9	10	.3333+02	.2100+06	.2500+03	-.5480-04	-.1644-05
14	10	.4167+02	.2100+06	.2000+03	-.4373+03	-.1049+02
13	9	.4167+02	.2100+06	.2000+03	-.4373+03	-.1049+02

7	8	4	3	.2500+03	.2000+03	.8077+05	.1000+01	-.3585-10	-.2896-05

IDEALIZED PANEL STIFFENER LOADS

ROD	NOD.PTS	AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
7	8	.3333+02	.2100+06	.2500+03	.7017-05	.2105-06
3	4	.3333+02	.2100+06	.2500+03	.0000	.0000
8	4	.4167+02	.2100+06	.2000+03	.2186+04	.5247+02
7	3	.4167+02	.2100+06	.2000+03	.2186+04	.5247+02

11	12	8	7	.2500+03	.2000+03	.8077+05	.1000+01	-.9101-10	-.7351-05

IDEALIZED PANEL STIFFENER LOADS

ROD	NOD.PTS	AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
11	12	.3333+02	.2100+06	.2500+03	-.6216-05	-.1865-06

7	8	.3333+02	.2100+06	.2500+03	.7017-05	.2105-06
12	8	.4167+02	.2100+06	.2000+03	.1312+04	.3148+02
11	7	.4167+02	.2100+06	.2000+03	.1312+04	.3148+02

15 16 12 11 .2500+03 .2000+03 .8077+05 .1000+01 -.815-09 -.1466-04

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
15	16	.3333+02	.2100+06	.2500+03	.9769-05	.2931-06
11	12	.3333+02	.2100+06	.2500+03	-.6216-05	-.1865-06
16	12	.4167+02	.2100+06	.2000+03	.4373+03	.1049+02
15	11	.4167+02	.2100+06	.2000+03	.4373+03	.1049+02

13 14 16 15 .2500+03 .8000+02 .8077+05 .1000+01 -.1286-09 -.1039-04

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
13	14	.1333+02	.2100+06	.2500+03	-.4234-04	-.3176-05
15	16	.1333+02	.2100+06	.2500+03	.3908-05	.2931-06
14	16	.4167+02	.2100+06	.8000+02	-.3272+03	-.7853+01
13	15	.4167+02	.2100+06	.8000+02	-.3272+03	-.7853+01

9 10 12 11 .2500+03 .8000+02 .8077+05 .1000+01 .1895-09 .1531-04

IDEALIZED PANEL STIFFENER LOADS

ROD NOD.PTS		AREA	E-VALUE	LENGTH	AXIAL LOAD	STRESS
9	10	.1333+02	.2100+06	.2500+03	-.2192-04	-.1644-05
11	12	.1333+02	.2100+06	.2500+03	-.2486-05	-.1865-06
10	12	.4167+02	.2100+06	.8000+02	.0000	.0000
9	11	.4167+02	.2100+06	.8000+02	.0000	.0000

```
 5    6    8    7    .2500+03    .8000+02    .8077+05    .1000+01    .7402-10    .5979-05
              IDEALIZED PANEL STIFFENER LOADS

 ROD NOD.PTS     AREA       E-VALUE      LENGTH      AXIAL LOAD     STRESS

      5    6    .1333+02    .2100+06    .2500+03    .4726-05    .3545-06
      7    8    .1333+02    .2100+06    .2500+03    .2807-05    .2105-06
      6    8    .4167+02    .2100+06    .8000+02    .2037-03    .4889-05
      5    7    .4167+02    .2100+06    .8000+02    .2037-03    .4889-05


@FIN                                                                    @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UNIVERSITY OF STELLENBOSCH

DEPARTMENT OF MECHANICAL ENGINEERING

FINITE ELEMENT ANALYSIS OF A WING TYPE STRUCTURE WITH EXPERIMENTAL VERIFICATION OF RESULTS

PART B

EXPERIMENTAL

By

E.M.E. BAUMGARTNER

Promoted by

Mr E. FOURIE

Prof. R.J. DU PREEZ

Second part of the thesis presented for the Degree of Master of Engineering

STELLENBOSCH JUNE 1976

ACKNOWLEDGEMENTS

I wish to thank the Mechanical Engineering Department's Technical Staff for their invaluable help in the construction of the structure and supports, especially to Mr H. Foot who did the lion's share of the work.

TABLE OF CONTENTS

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B. EXPERIMENTAL

B1 INTRODUCTION

The aim of the experimental work is to determine the degree of accuracy of the Finite Element Displacement Method applied to semi-monocoque structures.

A light three spar wing type structure is loaded under statically determinate and indeterminate external conditions. Strains are measured on the structure and a graphical representation of the results are made for direct comparison between theory and experiment.

B2 DESCRIPTION OF THE STRUCTURE

§ B2.1 Construction

A symmetrically tapered three spar box shaped structure was chosen for ease of construction in our workshops and for ease of geometrical design.

Figure B2.1.1 shows the general layout and shape of the structure.

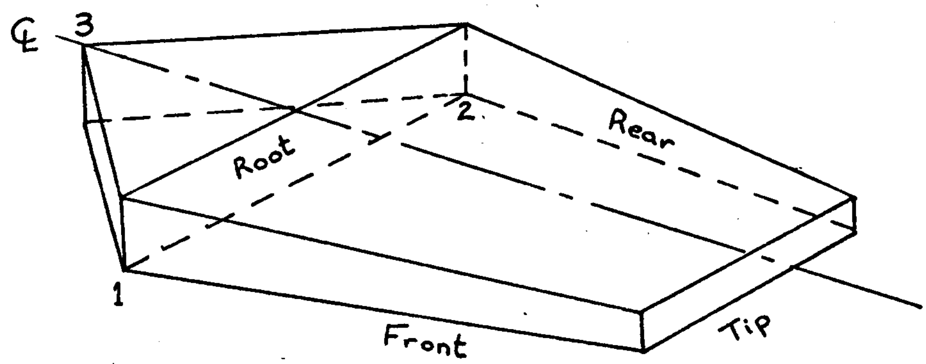


Figure B2.1.1 General layout of Structure

It is supported at points 1, 2 and 3 and loaded at various points near the tip. A centre spar runs straight down the centre line while the front and rear spars close the structure to form a typical 2 cell wing type section. The top and bottom is covered with a 0.7 mm thick aluminium skin (2S - ½H (H4)). The centre spar is an assembled I-beam having a 0.9 mm thick aluminium web (2S - ½H (H4)), with the top and bottom flanges made up of two ½" x ½" x $\frac{1}{16}$" aluminium angles (50S- TF) placed back to back and glued and pop rivetted to the web. The front and rear spars are of similar construction except that only one angle is used top and bottom. Evenly spaced ribs maintain the structure shape between the tip and the root. They are made of 0.9 mm aluminium sheeting (2S - ½H (H4)) and have two 50 mm ϕ holes punched and dished into them. The ribs are folded over on three sides, i.e. top, bottom and next to the front or rear spars and there glued and pop riveted to the adjacent structure. The top and bottom skin panels, between the spars, are stiffened by a $\frac{1}{2}" \times \frac{1}{2}" \times \frac{1}{16}"$ angle running along a line bisecting the distance between the spars. The spar webs are stiffened by the same size angles directly opposite the rib attachment points and act as the rib attachment to the centre spar.

A typical section through the structure is shown in Figure B2.1.2

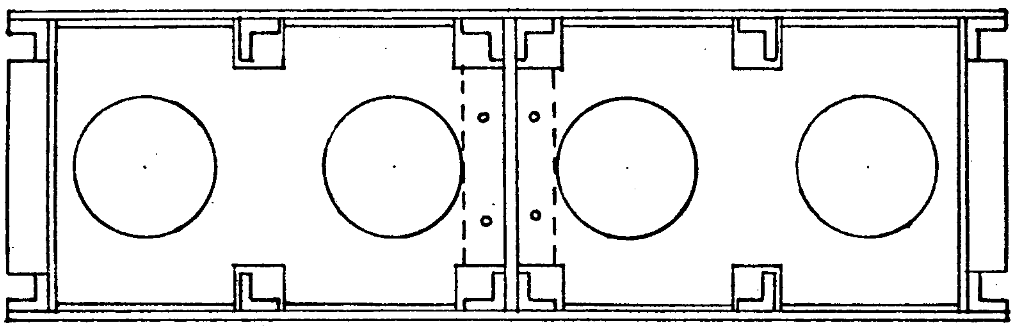


Figure B2.1.2 General View of Section through Structure

The glue used was Midbond 10 Epoxy Adhesive with an appoximate cured shear strength of 29 M.Pa. After each stage of the construction was completed the adhesive was allowed to cure for 12 to 15 hours at 35°C.

Photographs of both the completed structure and without the top skin can be seen in Appendix B1. The workshop drawings submitted are also included in the same appendix.

§ B2.2 Computer Model and Experimental Procedure

For analysis with the AIRSTR program, the spar caps, web stiffeners and rib folds were assumed to be endload or flange elements. The skin and rib panels were assumed to be plane stress membrane elements while the spar webs were assumed to be idealized panel elements.

Best results are obtained if the actual (experimental) strain readings are taken in the middle of the elements chosen. This eliminates to a great extent, the chance of stress concentrations affecting the readings. Rosette strain gauges (45^0) were attached to both sides of the centres of 5 top skin panels, 5 bottom skin panels and the 5 web panels between the root and the tip. Linear gauges were attached to the spar caps of the rear spar, centre spar and to the front skin stiffeners both top and bottom.

The strain gauges were split up into batches of 30 as the number of ballancing units was restricted to 30.

The top skin, bottom skin and main spar web, each having 5 panels with 3 component Rosettes on either side of the panels, were lumped together to give 30 gauges each,while the linear gauges were analysed together.

The instrumentation for the experimental readings was set up as shown in Figure B2.2.1. The linear gauges used on the structure were alumi= nium temperature compensating KYOWA KFD-5-C1-23 of resistance 120 Ω (nominal) and gauge factor 2.14. The rosettes used were Aluminium temperature compensating KYOWA KFC-5-D17-23 of resistance 120 Ω (nominal) and a gauge factor of 2.12. A single linear gauge of the first type was used as the dummy gauge. This was fixed to a piece of aluminium sheet of the type used for the webs and skin of the struc= ture.

Before commencement of each set of readings the Hottinger signal amplifier was callibrated with the calibration box to give a reading of 2V = 200 μ (1 μ = 10^{-6} m/m) for a gauge factor of 2. Recorded readings were thus adjusted by a factor of 2/gauge factor to give the actual strain in the structure.

It was discovered that the Digital Recording unit, when switched on, induced a voltage of 14 mV into the bridge circuit. Thus when calli= brating and zeroing the strain gauges all components of the equipment

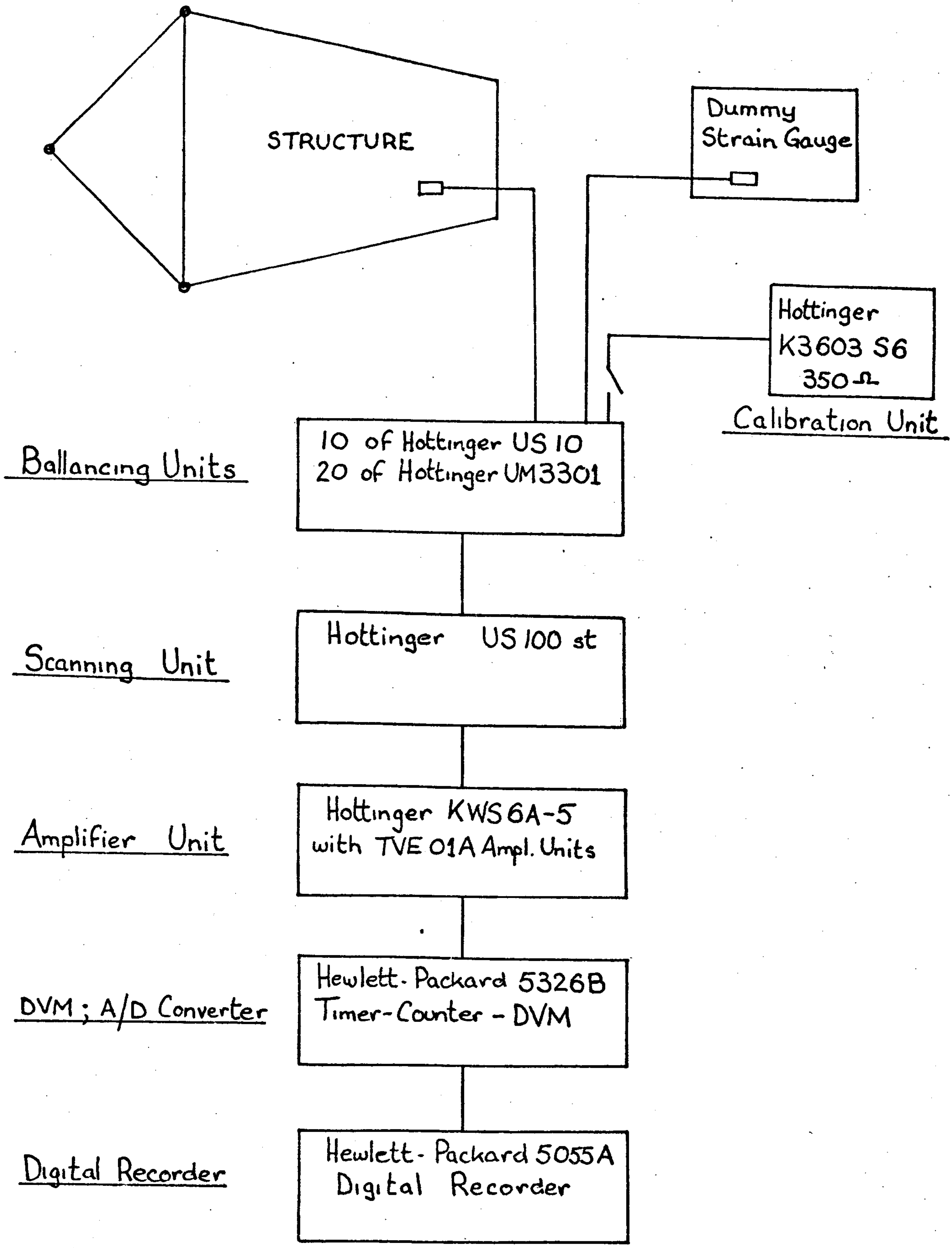

STRUCTURE
Dummy Strain Gauge
Hottinger K3603 S6 350 Ω
Calibration Unit
Ballancing Units
10 of Hottinger US 10
20 of Hottinger UM3301
Scanning Unit
Hottinger US 100 st
Amplifier Unit
Hottinger KWS 6A-5
with TVE 01A Ampl. Units
DVM ; A/D Converter
Hewlett-Packard 5326B
Timer-Counter - DVM
Digital Recorder
Hewlett-Packard 5055A
Digital Recorder

Figure B2.2.1

were allowed to warm up for 30 minutes and left on for the duration of the experiment.

The D.V.M.'s purpose was two-fold. Firstly to act as an analogue to digital converter of the bridge signal and secondly to act as an accurate means of reading off the zero point when ballancing the strain gauges, because the Hottinger Amplyfier Monitoring Gauge could only be read off to an accuracy of 20 mV.

Each batch of 30 gauges were ballanced and zeroed and then the readings for the 4 loading cases taken. Because of the tendency for the bridges to drift and the gauges to creep, a zero reading was taken before each loading case was applied to the structure. The difference between the readings then represented the strain in the structure.

The loads were applied to the structure by means of two "G" shaped frames which hung horisontally when a 4,54 kg mass was put onto the mass-pan. A drawing of the loading G can be seen in Figure B2.2.2.

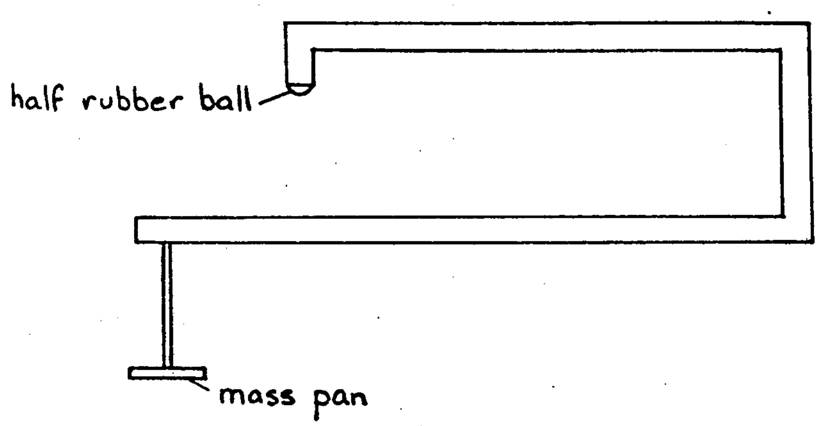


Figure B2.2.2 Loading "G"

The mass hung underneath the structure while the rubber ball pressed on the structure at the point required, giving an accurate point load. The loads were kept small so as to avoid buckling in the underside skin panels as much as possible.

To accomplish the theoretical analysis of the structure it was broken up into elements corresponding, as near as possible, to that of the actual structure. The division of the structure and the nodal point numbering is shown in Figure B2.2.3

The elements singled out for comparison were

(i)	Top skin panels:	12	10	40	42
		10	8	38	40
		8	6	36	38
		6	4	34	36
		4	2	32	34
(ii)	Bottom skin panels:	11	9	39	41
		9	7	37	39
		7	5	35	37
		5	3	33	35
		3	1	31	33
(iii)	Main spar web panels:	69	67	68	70
		67	65	66	68
		65	63	64	66
		63	61	62	64
		61	59	60	62
(iv)	Spar caps and stiffe= ners				
	Rear spar top flange:		20	22	
			22	24	
			24	26	
			26	28	
			28	30	
	Rear spar bottom flange:		19	21	
			21	23	
			23	25	
			25	27	
			27	29	

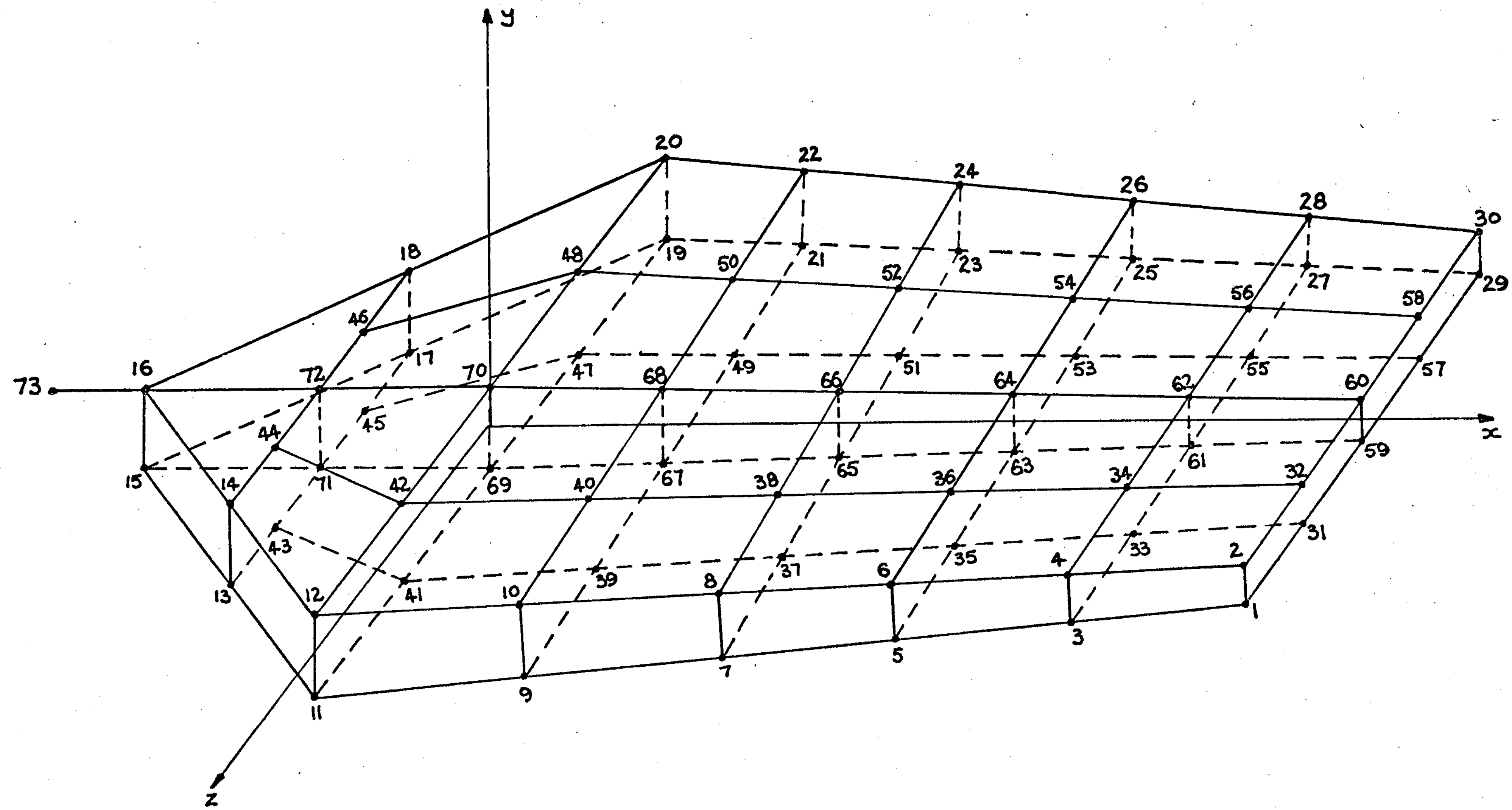


Figure B2.2.3 Nodal point numbering

Main spar top flange:	60	62
	62	64
	64	66
	66	68
	68	70
Main spar bottom flange:	59	61
	61	63
	63	65
	65	67
	67	69
Top skin stiffener:	32	34
	34	36
	36	38
	38	40
	40	42
Bottom skin stiffener:	31	33
	33	35
	35	37
	37	39
	39	41

Two basic loading cases were used, one symmetrical and the other assymmetrical. Both were applied to the structure for twc different support restriction cases, i.e.

(i) Free and

(ii) Fixed.

Free implies that the supports at nodal points 11 and 19 were free to move in the z-direction while 16 was free in the x-direction.

Fixed implies that the supports 11 and 19 were restricted in all 3 global co-ordinate directions while at 16, because of the bending effect of the rear support, it was not absolutely rigid in the x-direction. An equi= valent endload element was inserted aft of nodal point 16 (16 - 73) and given such dimensions as to be equivalent to the bending effect of the rear supporting structure.

The structure was thus tested for the following 4 conditions:

Loading Case 1	(1)	Free	(LC = 1 Free)
	(2)	Fixed	
Loading Case 2	(3)	Free	
	(4)	Fixed	

For Loading Case 1 Weights of
67,21 N at nodal point 58
and 66,92 N at nodal point 32 were applied to the structure (neg. y-direction)

For Loading Case 2 Weights of
67,21 N at nodal point 2
and 66,92 N at nodal point 34 were applied to the structure (neg. y-direction)

The actual dimensions of the elements were used wherever they coincided with the nodal points. The nodal points did not however coincide with the centres of gravity of the spar caps and stiffeners so a correction to the areas was made to ensure that the moment of inertia of the section remained correct. An effective thickness was given to the ribs to allow for the lightening holes cut into them.

A graphical comparison of the results is given in Appendix B2.

B3 DISCUSSION OF RESULTS

From the computer model prodicted stresses, it was seen that the maximum strains expected were in the order of 30 μ (30 x 10^{-6} m/m). This meant that a high amplification setting had to be used on the bridge measuring equipment, i.e. 200 μ = 2 V. This implies an extremely sensitive instal= lation in that 10 mV of drift on the bridge means 1 μ error in the reading which means a 3% error for every 1 μ of drift.

Initially using the QUA4 membrane element (linear displacement field, straight sided quadrilateral) for all the quadrilateral panels and a uniform Elasticity Modulus for the Aluminium (panels and flanges), a discrepancy of 25% and worse in the flanges was recorded between expected and actual strains. It was suspected that localized bending effects were causing the additional strain in the experimental results.

Using the ASKA program, eccentrically loaded beam elements were compared with flange elements in a similar type of structure. The difference in results was negligible and the error sought elsewhere.

Enquiries directed at the manufacturers of the material revealed that the Elasticity Modulus of the sheet material was some 7% higher than that of the angles used and the Poisson's ratio 13% higher than initially assumed. An experiment carried out on the flange material confirmed the manufacturers value, using these values a new computer prediction for the strains was obtained. These predicted values can be seen in the graphs of Appendix B2 labeled EXACT.

It was still felt however that the correlation should be better.

In Reference 3 (Part A), mention is made of the bad behaviour of the linear plane stress quadrilateral (QUA4) under pure bending. In the derivation thereof, the assumption is made that the panel sides remain straight and do not curve, as would be expected under pure in plane bending. This lead to the conclusion that because the element is stiffer in bending than it should be, the predicted loads in the spar flanges and stiffeners would be lower than in reality. Now under pure bending the Idealized Panel Element conforms exactly to the Elementary Theory of Bending (Reference 1, Part A), so these elements were inserted in place of the QUA4 elements in the webs of the 3 spars. The strain pre= diction in the spar caps and stiffeners was greatly improved as can be seen in Appendix B2, Graphs 1 to 6, 12 to 17, 23 to 28 and 34 to 39 curves labeled MIXED. The upper prediction line labeled ETB, on the fore mentioned graphs, was obtained by using Rod elements and computer generated idealized quadrilaterals and triangles. It is evident that this model is wholely inadequate and more careful modelling is required, by including the whole skin cross sectional area into the shear panel stiffeners.

Comparing the MIXED prediction and Experimental results of the lower skin stiffener, (Graphs 4, 15, 26, 37) it can be seen that the stringer loads between the front supports (Nodal points 11 and 19) and 70% of the span, tend to be higher than predicted. This was to be expected because the strain gauge readings on the bottom skin panels indicated that buckling had taken place and the skin was thus carrying less load than expected. This is born out by inspection of Graphs 8, 19, 30 and 41 in which the direct strains $\varepsilon_{\overline{xx}}$ are lower than those predicted by the MIXED model. The extreme deviation from the predicted strains near the supports (10% span) can be attributed firstly to greater buckling and secondly to the coarseness of the F.E. grid near the sup= ports which are in effect a discontinuity in the structure and in which localized effects play a major role.

The main spar flange loads, both top and bottom, Graphs 1, 2, 12, 13, 23, 24, 34 and 35, follow the predicted strains closely in the region 30 to 100% of the span. The deviation at the 10% span reading on the lower flanges can be attributed once again to the proximity of the supports and the probable buckling of the skin panels in that region. The good correlation can be attributed to the fact that the flanges are securely held by the surrounding structure, i.e. the spar web and the skin, and they thus follow very closely the approximation that they only carry axial stresses and remain straight.

Inspection of the top skin stiffener (stringer) results Graphs 3, 14, 25 and 36 indicate that the assumptions made in the computer model follow reality very closely. The maximum deviation being in the order of 2 μ which represents a 7,5% error. Considering the sensitivity of the data acquisition apparatus this is negligible.

The direct strain readings in the top skin panels, Graphs 7, 18, 29 and 40 show a maximum deviation of 12,5% in the results between 30% and 100% of the span. Support proximity effects appear to play a role in the panel nearest to it. The 12,5% maximum deviation in the results, can be attributed to the fact that the QUA4 element sides remain straight and do not curve and can therefore not follow the exact defor= mation of the structure. This deformation difference however, cannot be too great, as the results converge, towards the tip of the structure.

The rear spar flange results, Graphs 5, 6, 16, 17, 27, 28, 38 and 39 suggest that the Idealized Panel assumption is too severe and that an element somewhere between the QUA4 and the Idealized Panel should be used in modelling the spar webs. This could be accomplished by using a higher order element, i.e. one which would curve in bending as well as maintain it's plane stress membrane characteristics. The correlation is, however, still good over 70% of the span with the now characteristic rise in the experimental results near the supports.

The shear strain components of the top and bottom skin panels are plotted on Graphs 9, 10, 20, 21, 31, 32, 42 and 43. The large deviations in the bottom panels nearest the support in the fixed loading case conditions can be attributed to the fact that, in the theoretical model, a single panel cannot deform sufficiently to give a realistic prediction of the strains expected throughout the panel.

In a 45^{o} three component rosette all three direct strains are needed to predict the shear strain. From Mohrs strain circle we have

$$\varepsilon_{xy} = \varepsilon_{xx} + \varepsilon_{yy} - 2\,\varepsilon_{45}$$

It is clear that should there be an error in all three of the strain components and should they all be in the "wrong" direction, then the error is four times as large as for a single strain, using the same sensitive bridge setting.

It should be noted at this stage that the sign of the shear strain component given by the finite element analysis is opposite to that of the experimental results.

Graphs 11, 22, 33 and 44 show the results of the shear strains in the main spar web panels. The EXACT prediction in the fixed loading case conditions on the panel nearest the web is way off the mark; even the sign is incorrect. This is once again due to the limitations of the QUA4 element. The error between the MIXED prediction and the experimental results can be accounted for by the same reasoning as for the skin shear strains. The trends however are correct as is very clearly evident in Graphs 33 and 44 for the unsymmetrical loading case.

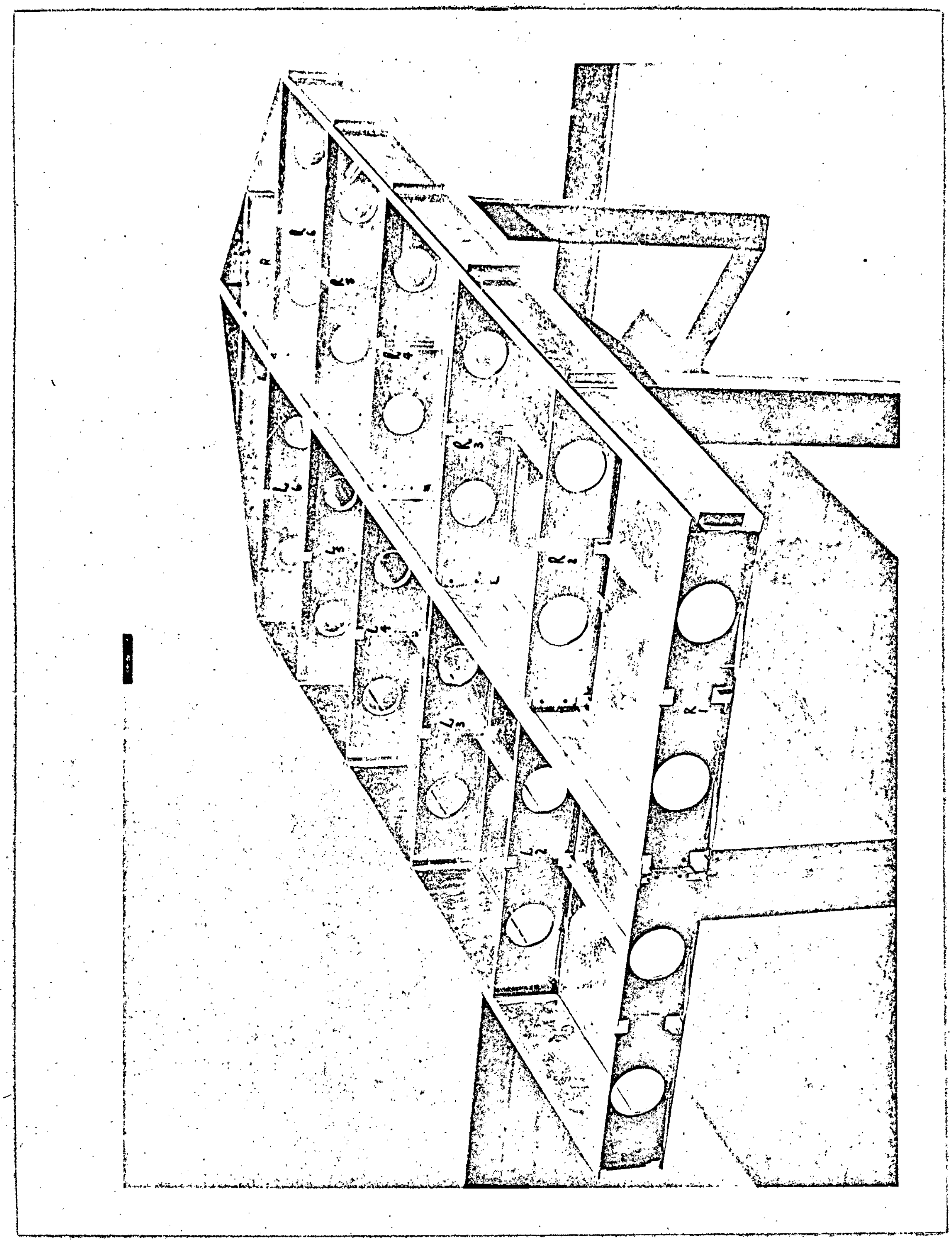
B4 CONCLUSIONS

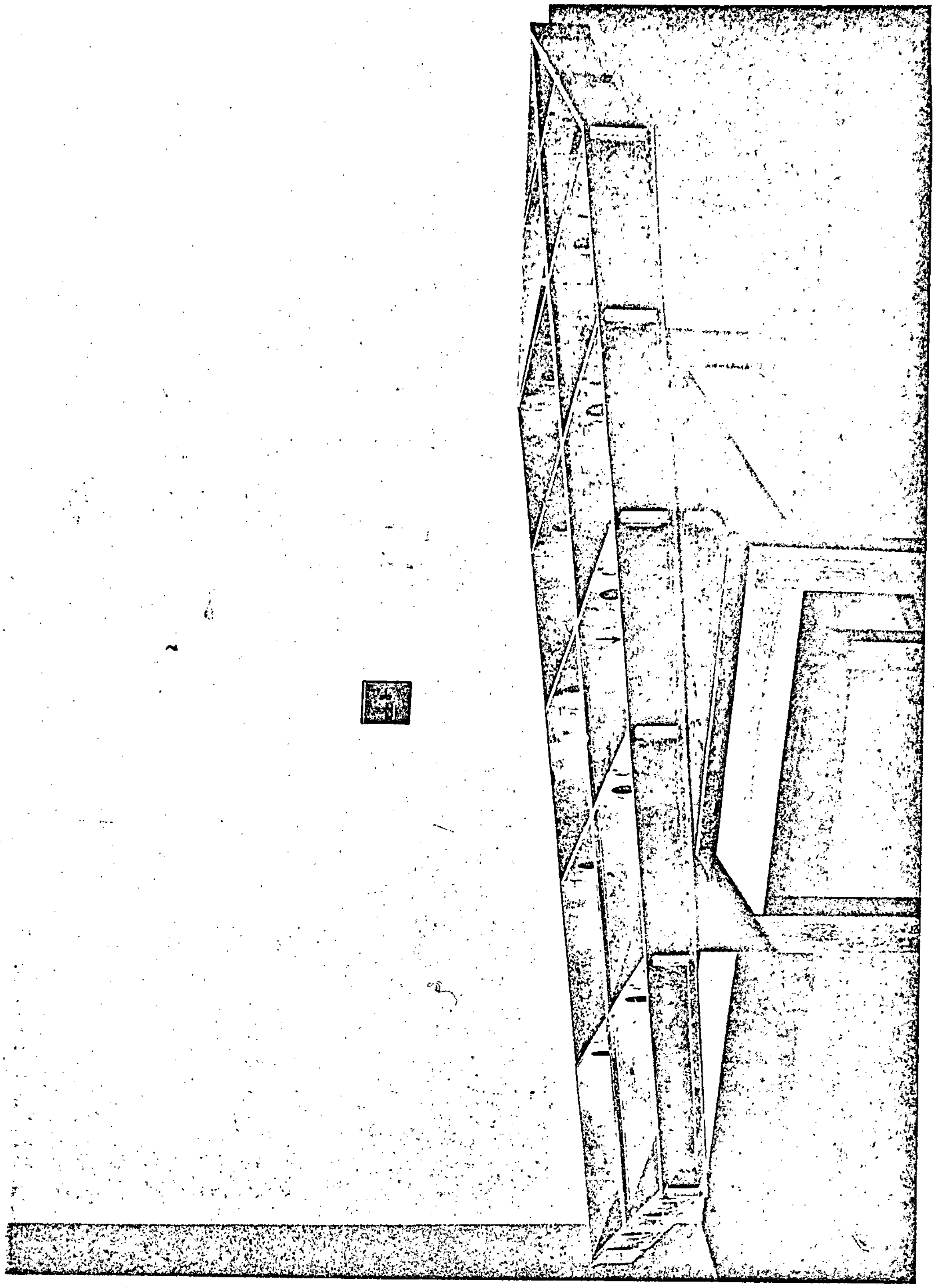
The results show that the finite element method can be used with confidence in predicting accurately the stresses in a complex structure. However, care and a knowledge of the limitations of the elements to be used, is of great importance.

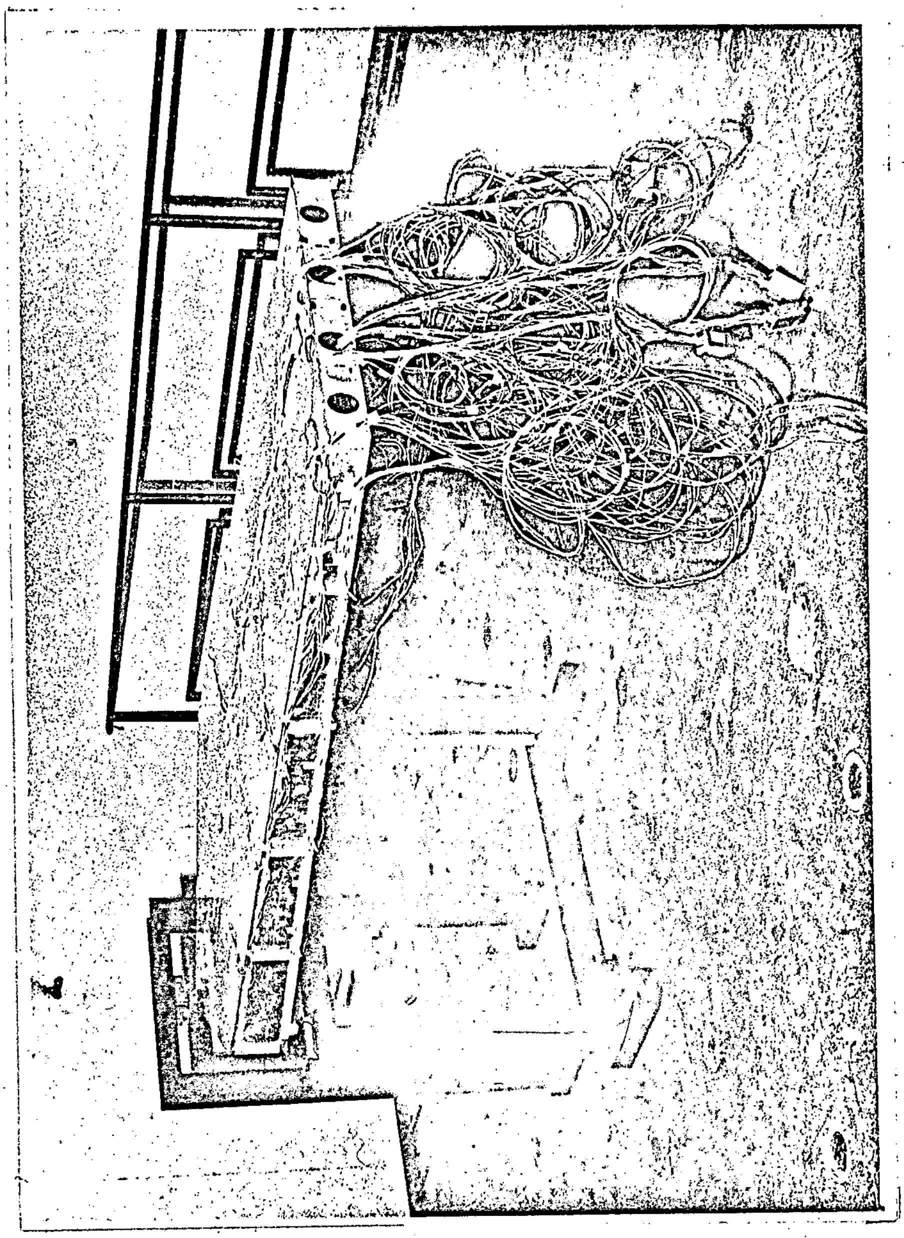
More sophisticated elements have been developed and are available in the large Finite Element Systems which are on the market. These higher order elements do however, take more computer time to assemble and result in larger sets of equations to solve, so it is ultimately up to the stress analyst to weigh up the advantages of greater accuracy against cost of analysis. It appears though that careful use of the simpler elements can give adequate accuracy for design purposes as the predicted strains are slightly higher than those actually measured.

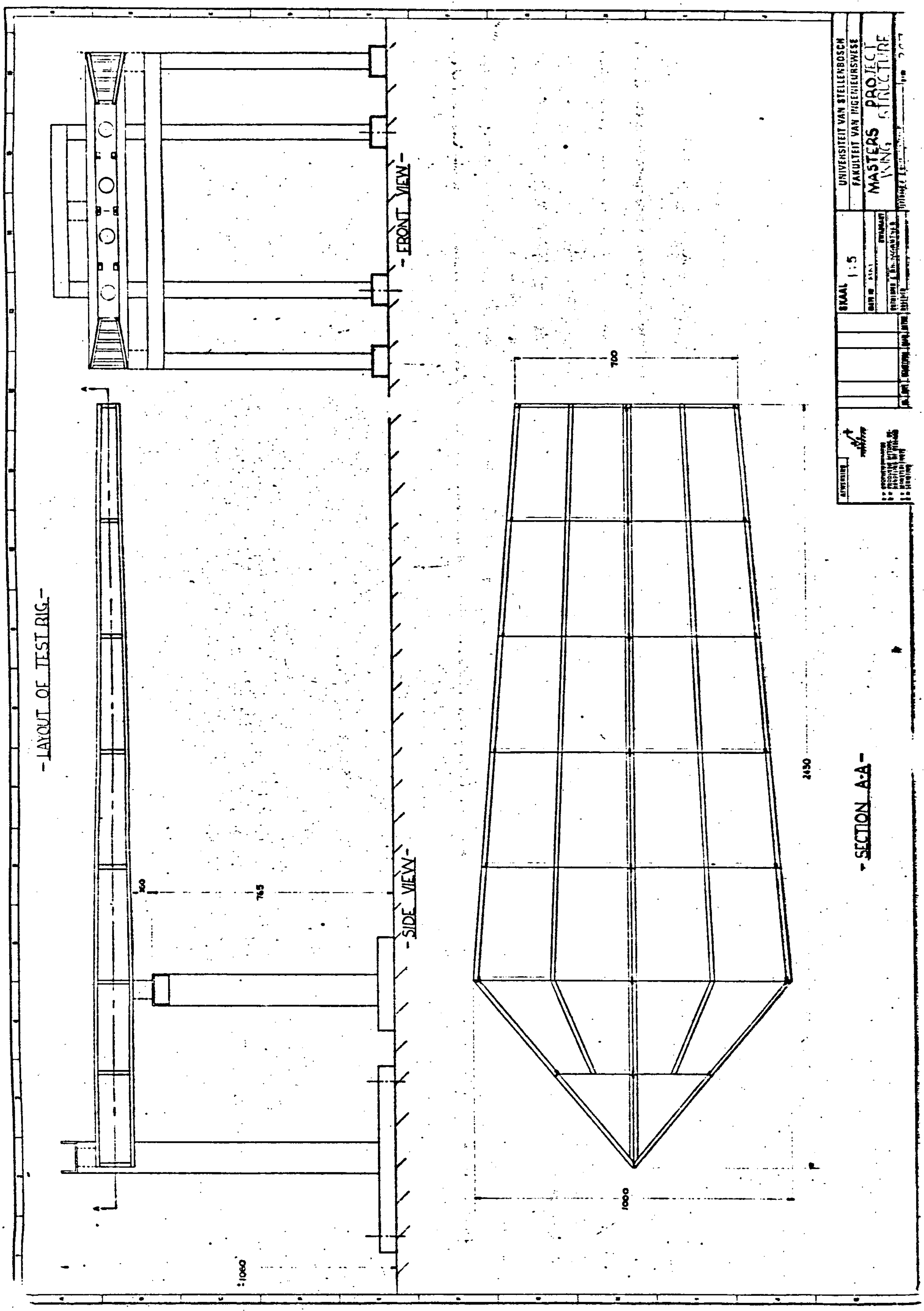
APPENDIX B1

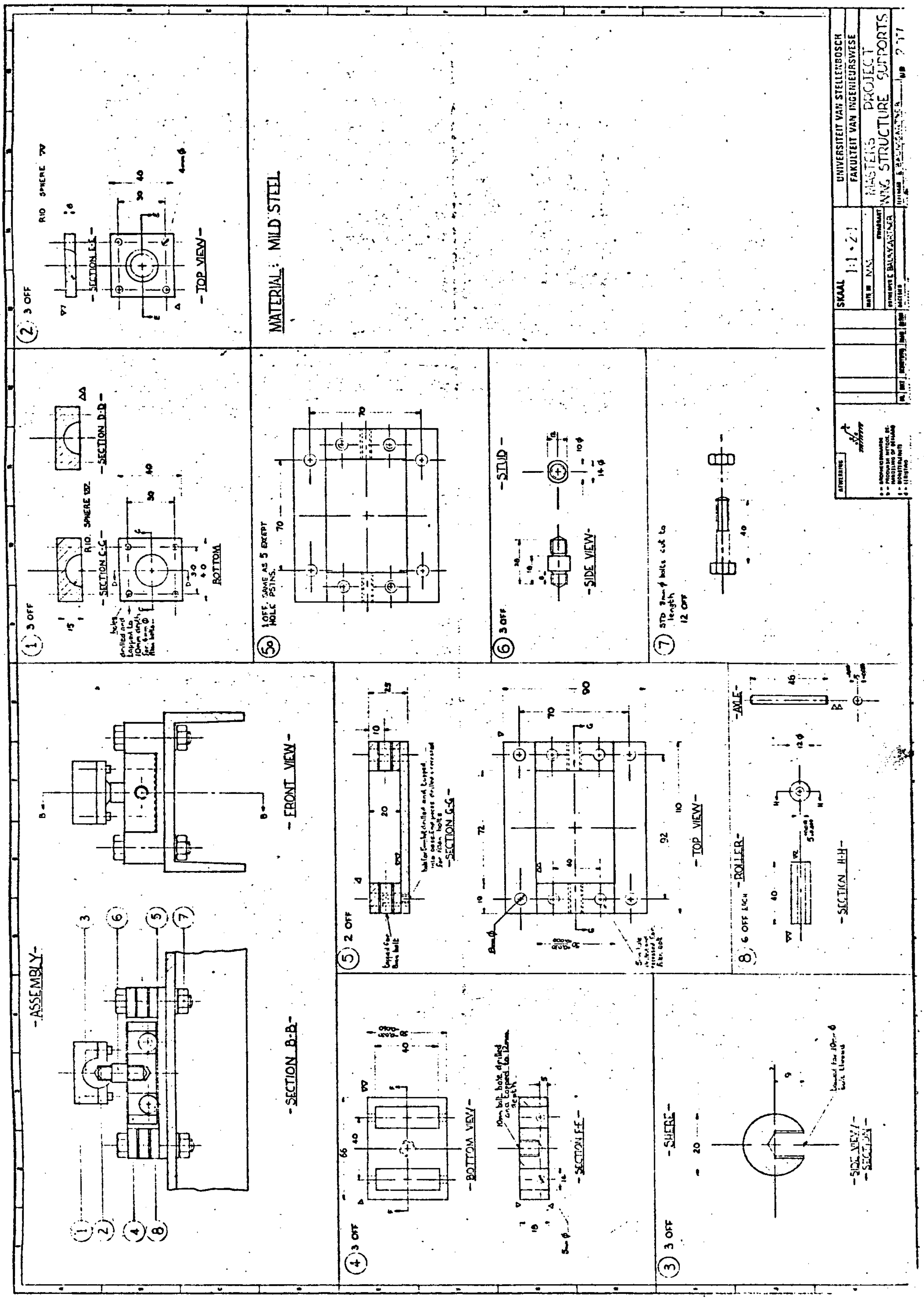
Photographs and Workshop drawings of test structure.



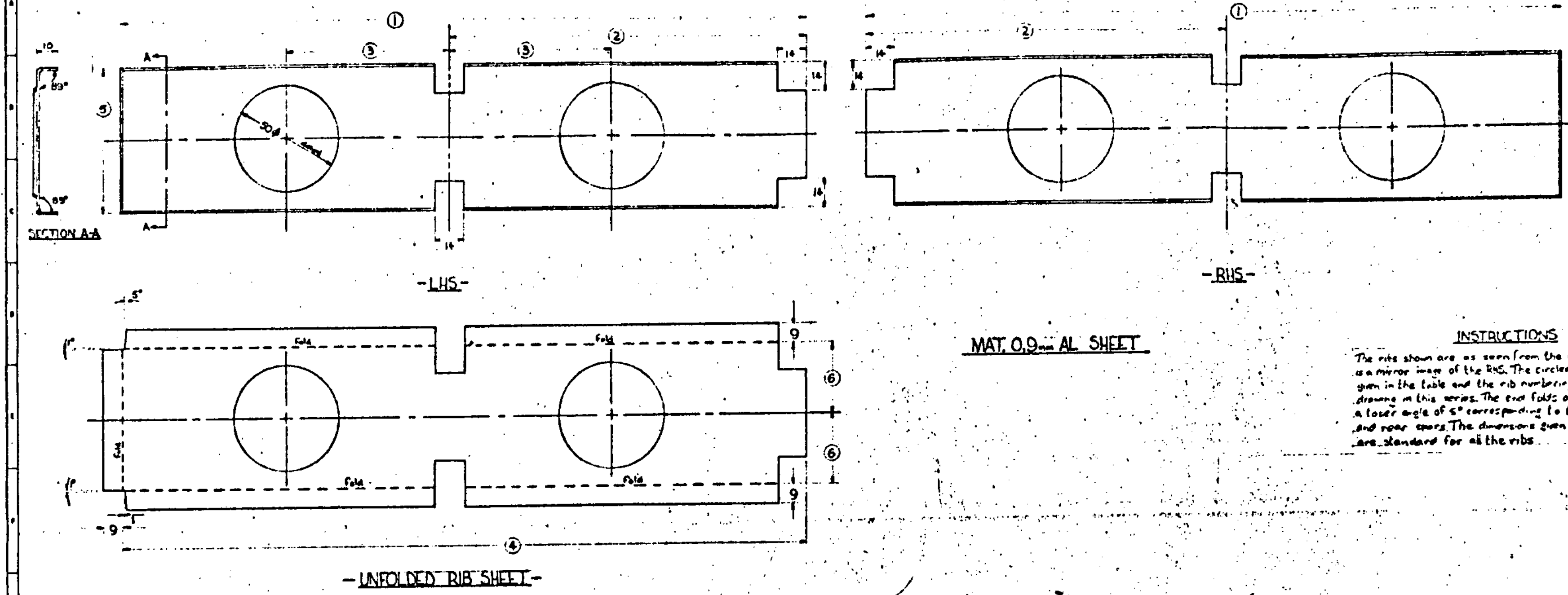





- LAYOUT OF TEST RIG -
- FRONT VIEW -
- SIDE VIEW -
- SECTION A-A -
700
2450
1000
765
100
1060
UNIVERSITEIT VAN STELLENBOSCH
FAKULTEIT VAN INGENIEURSWESE
MASTERS PROJECT
WING STRUCTURE
SKAAL 1:5


UNIVERSITEIT VAN STELLENBOSCH
FAKULTEIT VAN INGENIEURSWESE
MASTERS PROJECT
WING STRUCTURE SUPPORTS
SKAAL
1:1 • 2:1
MATERIAL: MILD STEEL
-ASSEMBLY-
-SECTION B-B-
-FRONT VIEW-
2 3 OFF
-SECTION E-E-
-TOP VIEW-
1 3 OFF
-SECTION C-C-
-SECTION D-D-
BOTTOM
5a 1 OFF, SAME AS 5 EXCEPT HOLE PSTNS.
6 3 OFF
-STUD-
-SIDE VIEW-
7 STD 8mm ϕ bolts cut to length 12 OFF
5 2 OFF
-SECTION G-G-
-TOP VIEW-
8 6 OFF EACH
-AXLE-
-ROLLER-
-SECTION H-H-
4 3 OFF
-BOTTOM VIEW-
-SECTION F-F-
3 3 OFF
-SHERE-
-SIDE VIEW/SECTION-

RIBS



MAT. 0,9mm AL SHEET

INSTRUCTIONS

The ribs shown are as seen from the wing tip. The LHS rib is a mirror image of the RHS. The circled dimensions are given in the table and the rib numbering given on the 3rd drawing in this series. The end folds on the ribs must have a lower angle of 5° corresponding to that of the front and rear spars. The dimensions given on this drawing are standard for all the ribs.

RIB DIMENSION TABLE

rib	1	2	3	4	5	6	
a	336	174,5	80	335	70	34	
b	366	189,5	87	365	81	39,5	
c	396	204.5	95	395	92	45	
d	426	219,5	103	425	103	50.5	
e	456	234,5	110	455	114	56	
f	486	249,5	117	485	125	61,5	
g	235	174,5	No [illegible]	234	125	61,5	End fold 40° [illegible] 5°

AFWERKING

SKAAL 1:1

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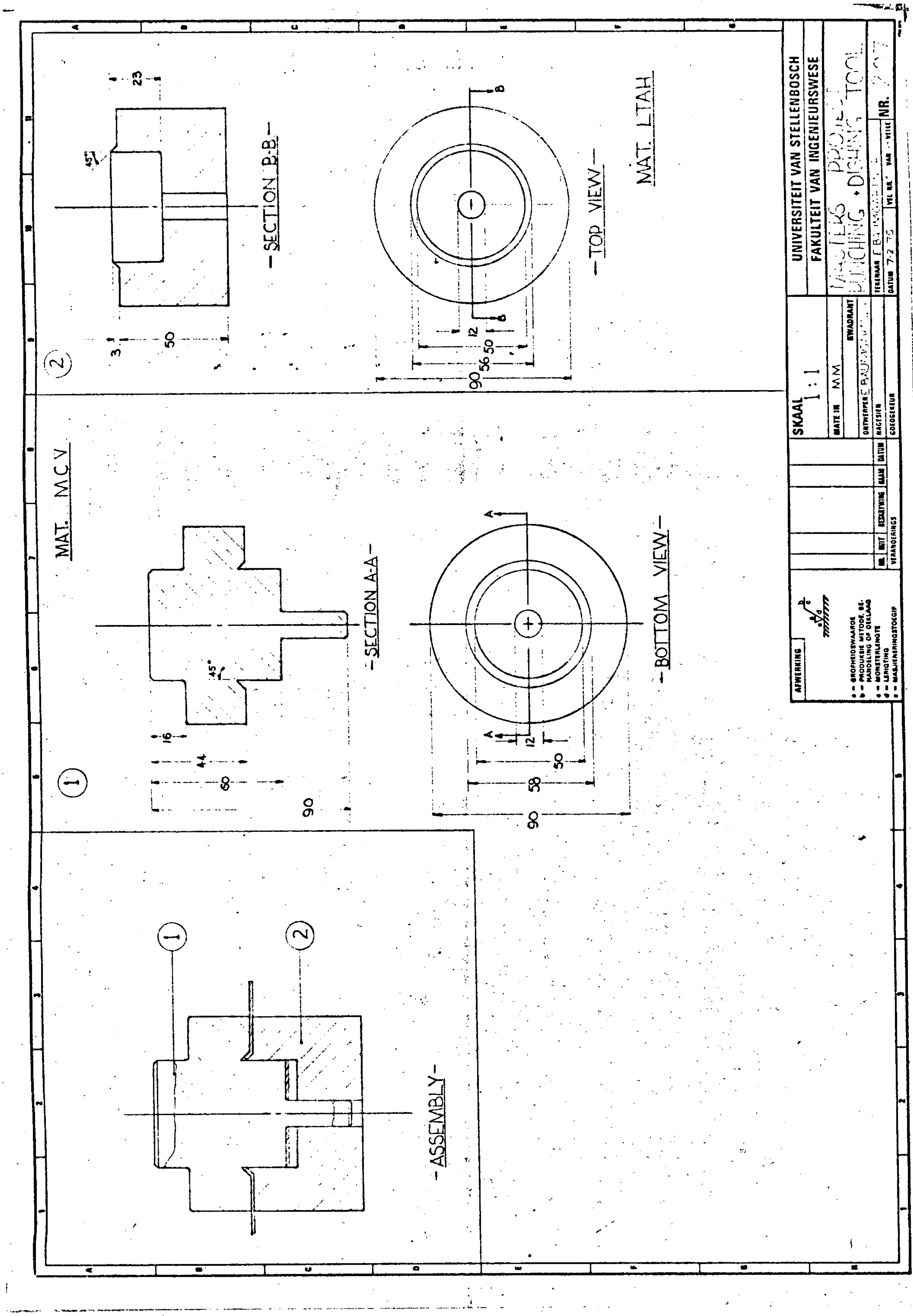
NR 207



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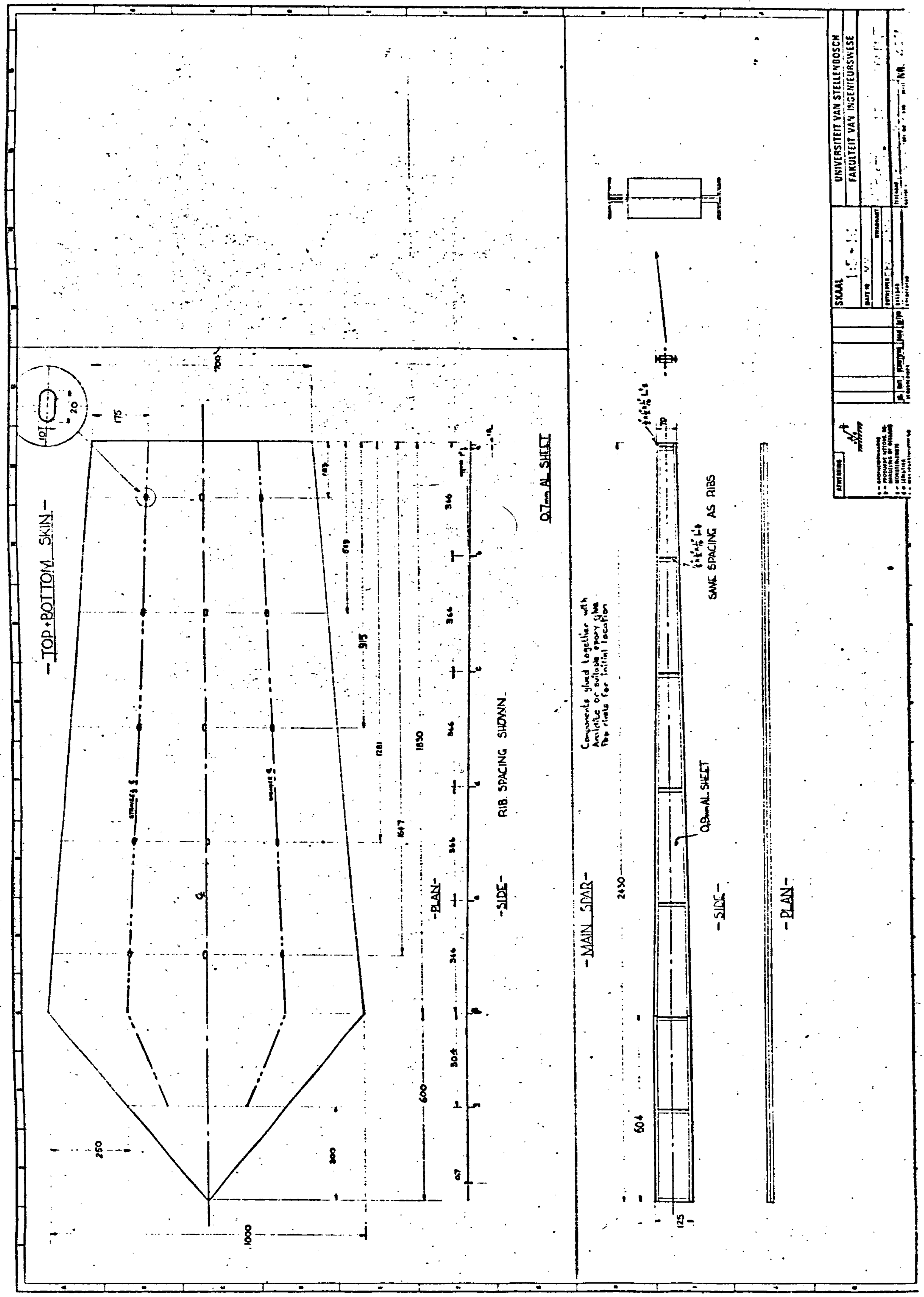
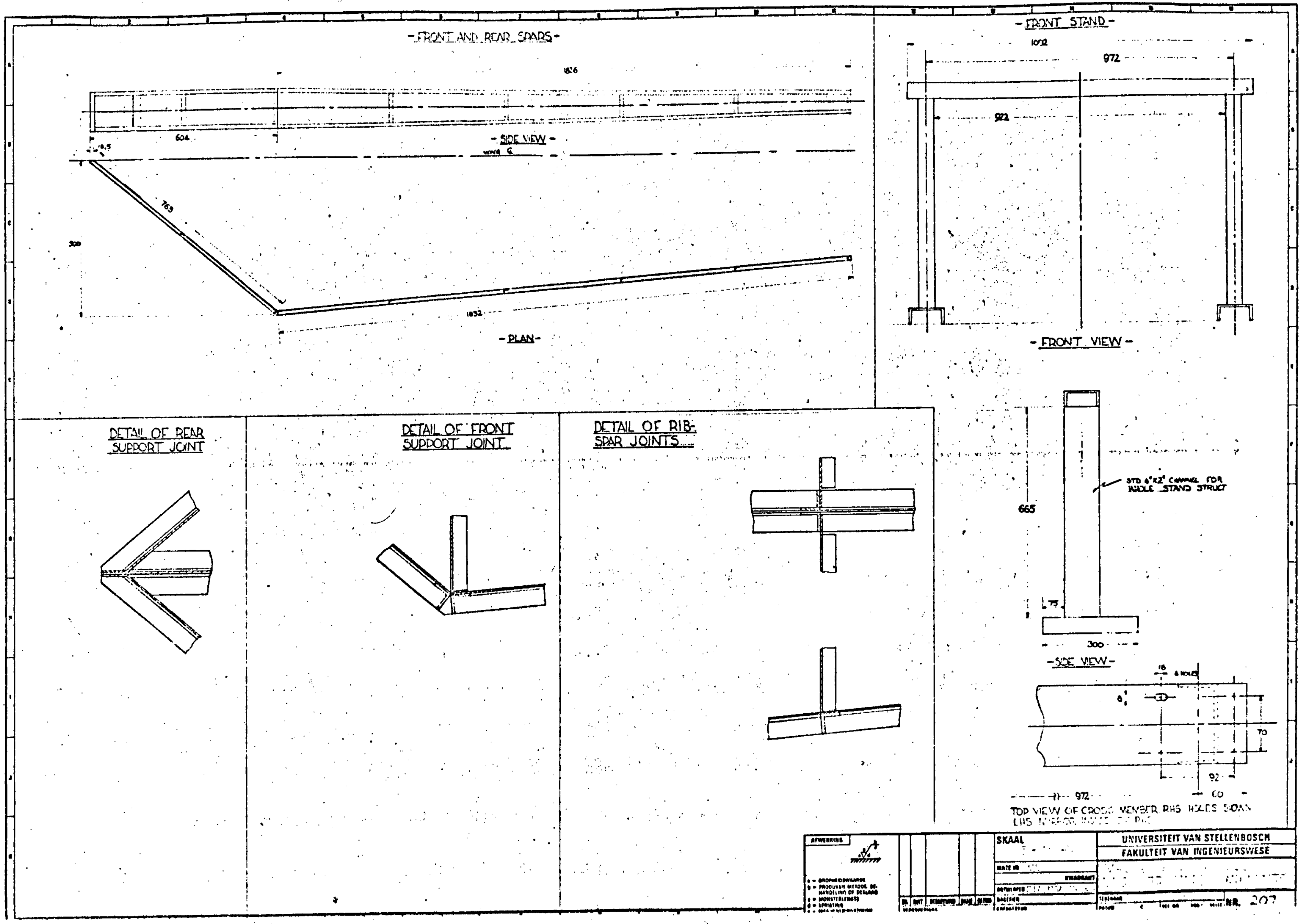
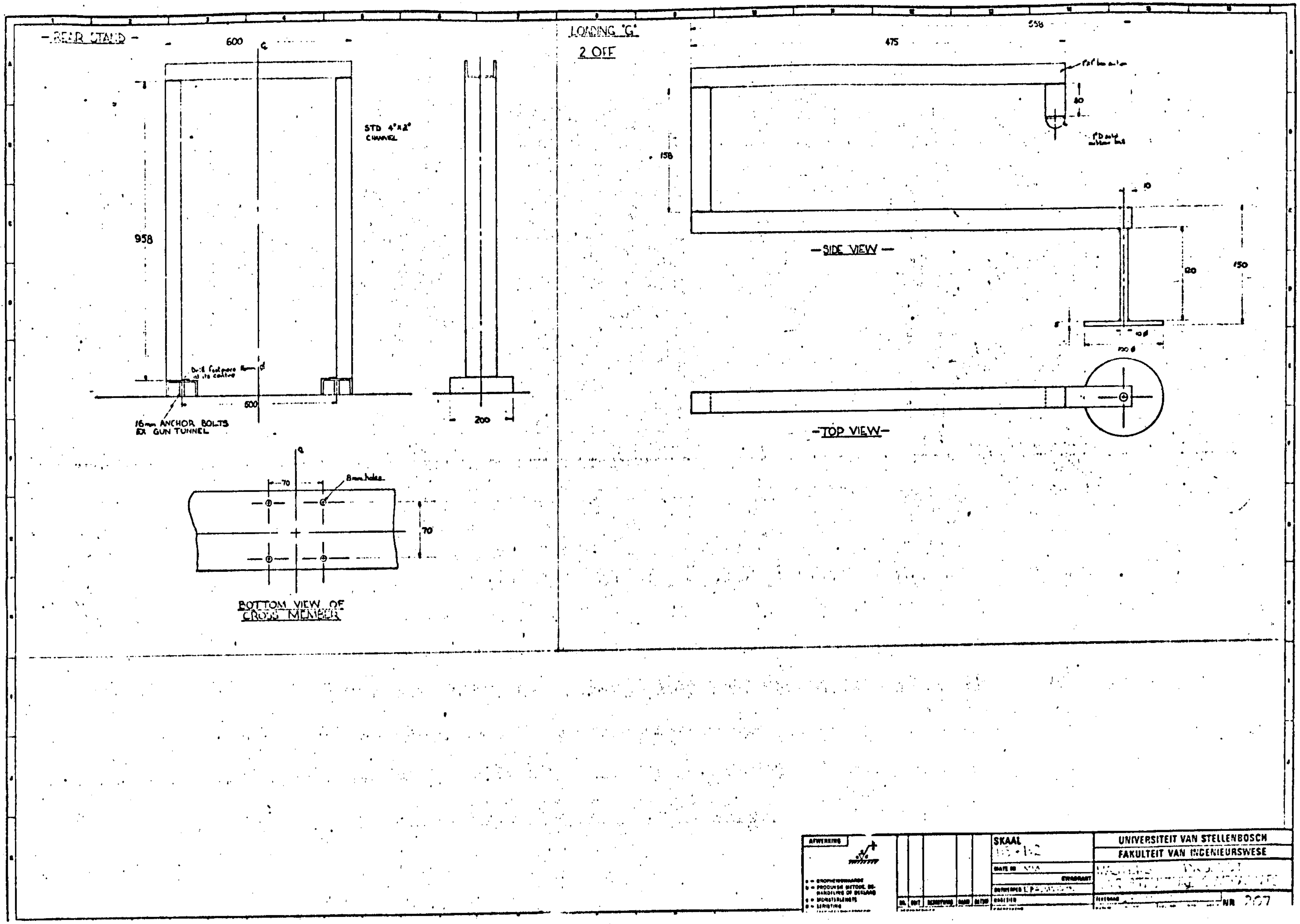
SKAAL 1:1
MATE IN MM
KWADRANT
ONTWERPER
TEKENAAR
DATUM 7-2-75
NR.
NAGESIEN
GOEDGEKEUR
VERANDERINGS
BESKRYWING
NAAM
DATUM
AFWERKING
GROFHEIDSWAARDE
PRODUKSIE METODE, BEHANDELING OF OORLAAG
MONSTERLENGTE
LIGRIGTING
MASJIENERINGSTOEGIF
SECTION B-B
TOP VIEW
MAT. LTAH
SECTION A-A
BOTTOM VIEW
MAT. MCV
ASSEMBLY
45°
23
50
3
8
12
56
50
90
16
44
60
58
A



-TOP + BOTTOM SKIN-
0.7mm AL SHEET
RIB SPACING SHOWN
-PLAN-
-SIDE-
-MAIN SPAR-
Components glued together with Araldite or suitable epoxy glue. Pop rivets for initial location
0.9mm AL SHEET
SAME SPACING AS RIBS
-SIDE-
-PLAN-
UNIVERSITEIT VAN STELLENBOSCH
FAKULTEIT VAN INGENIEURSWESE
SKAAL



- FRONT AND REAR SPARS -
1826
604
- SIDE VIEW -
763
500
1832
- PLAN -
- FRONT STAND -
1032
972
922
- FRONT VIEW -
STD 6"x2" CHANNEL FOR WHOLE STAND STRUCT
665
75
300
- SIDE VIEW -
70
92
60
972
TOP VIEW OF CROSS MEMBER RHS HOLES SHOWN
DETAIL OF REAR SUPPORT JOINT
DETAIL OF FRONT SUPPORT JOINT
DETAIL OF RIB-SPAR JOINTS
SKAAL
UNIVERSITEIT VAN STELLENBOSCH
FAKULTEIT VAN INGENIEURSWESE
NR. 207



- REAR STAND -
600
958
STD 4"x2" CHANNEL
600
16mm ANCHOR BOLTS EX GUN TUNNEL
200
8mm holes
70
70
BOTTOM VIEW OF CROSS MEMBER
LOADING 'G'
2 OFF
558
475
158
80
10
120
150
- SIDE VIEW -
- TOP VIEW -
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FAKULTEIT VAN INGENIEURSWESE
SKAAL
NR 207

APPENDIX B2

Graphical representation of predicted and experimental strains

ETB Prediction curve using computer generated Idealized Panels and Flange Elements.

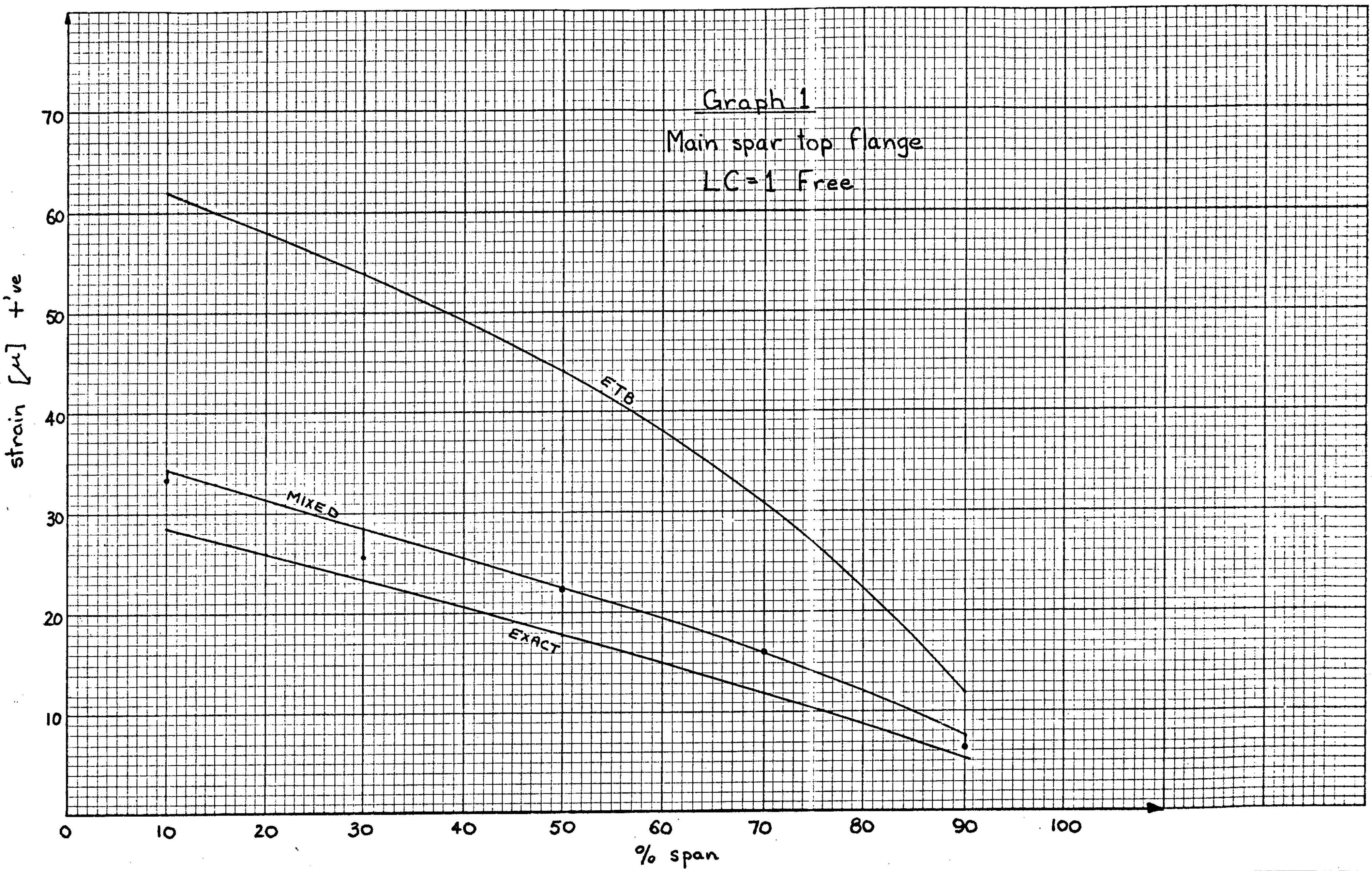
EXACT Prediction curve using Plane Stress Quadrilaterals, C.S.T. and Flange Elements.

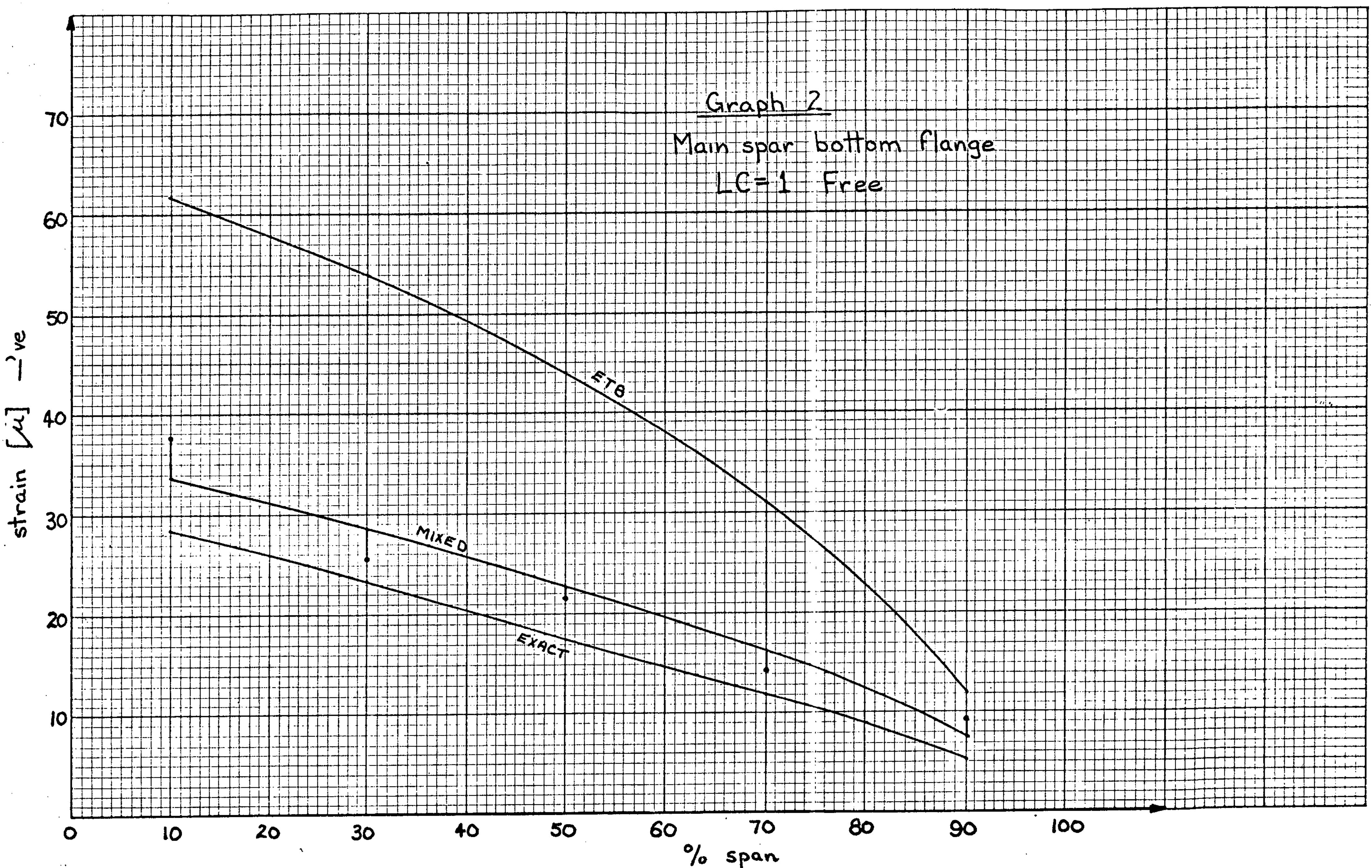
MIXED Prediction curve using Plane Stress Quadrilaterals and C.S.T. Elements for the skin and ribs, computer generated Idealized Panels for the spar webs, and Flange Elements.

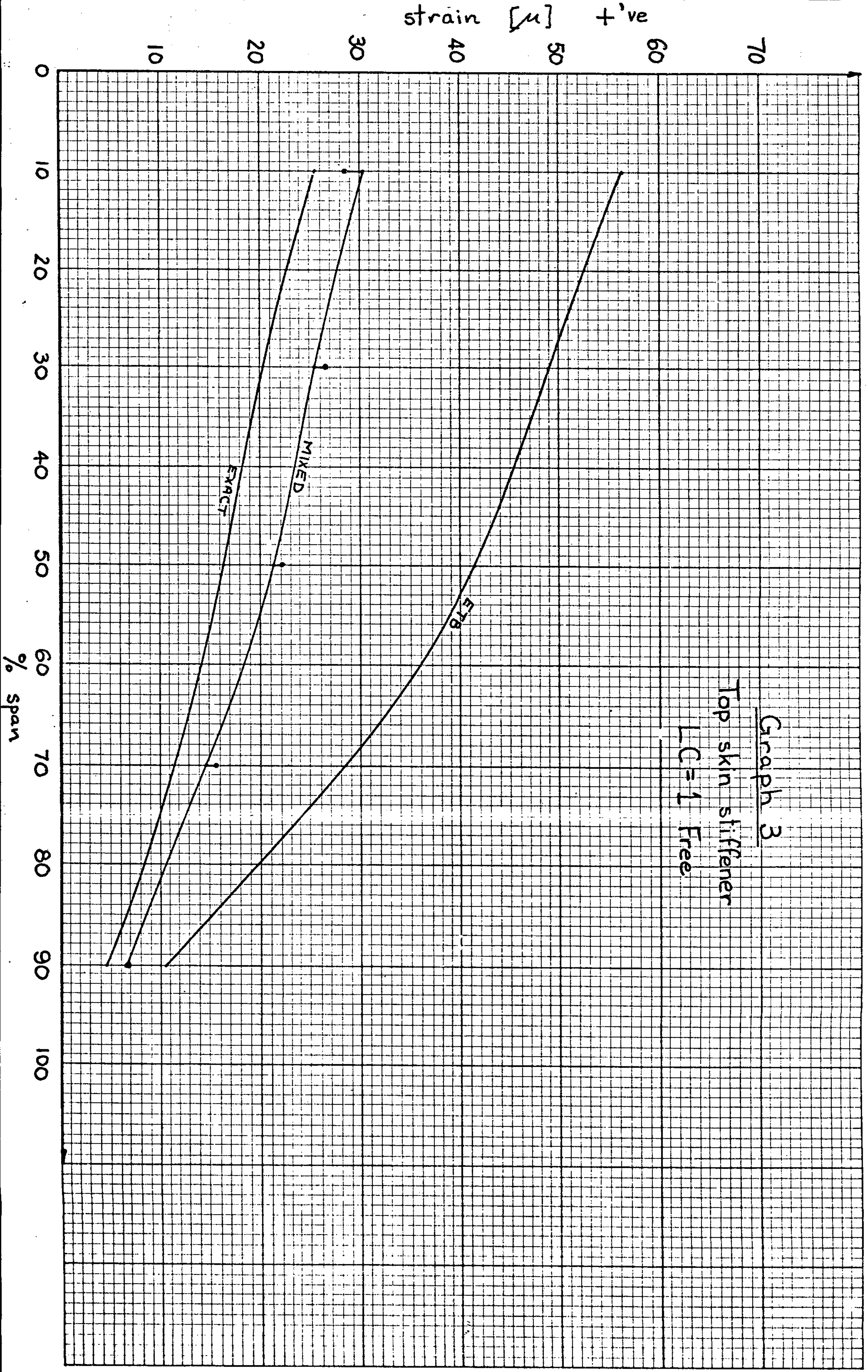
• Experimental results.

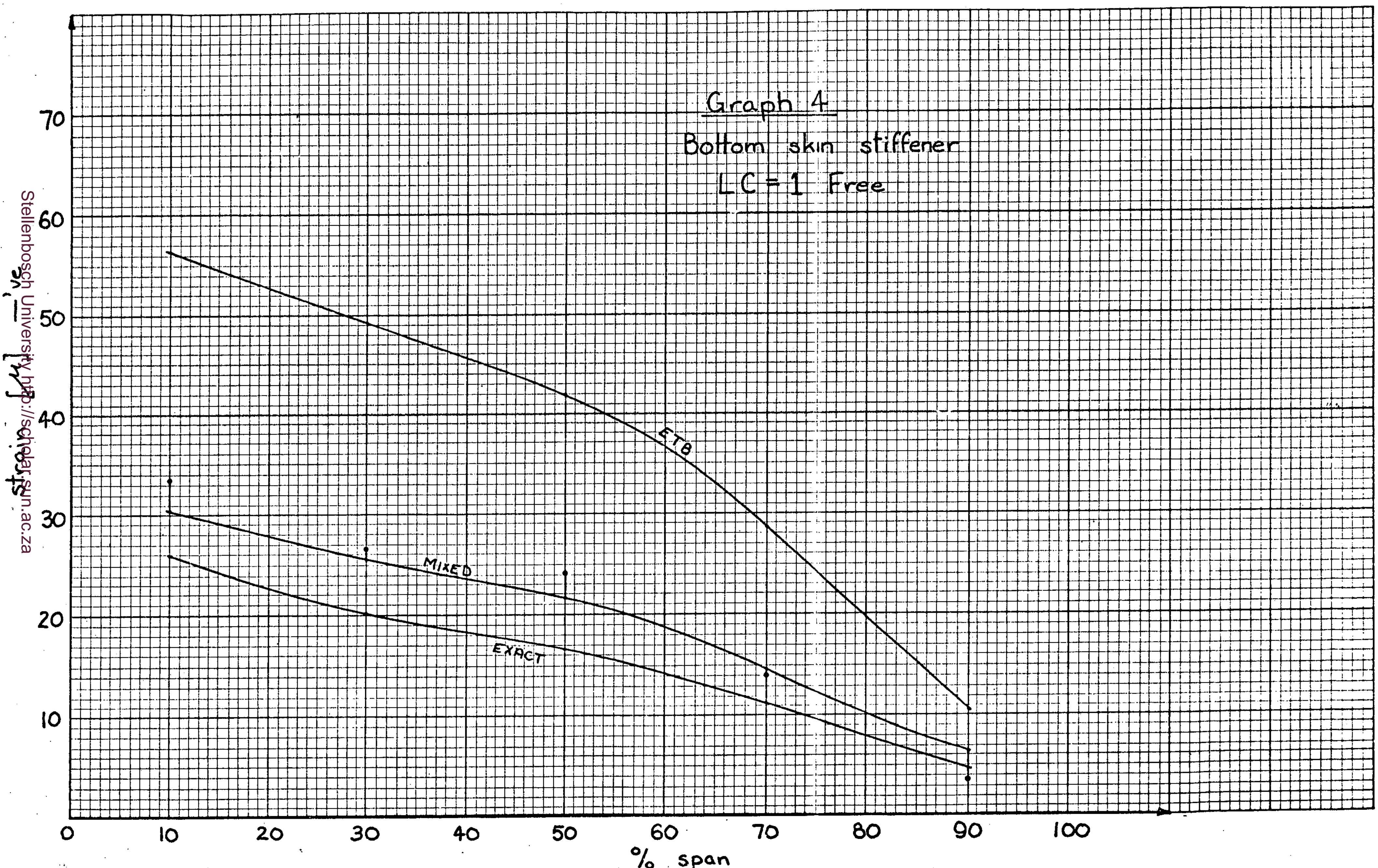
$$1\ \mu = 1 \times 10^{-6}\ \text{m/m} \quad \text{(strain)}$$

Note: The % span is taken from a line through the front supports to the free end of the structure.

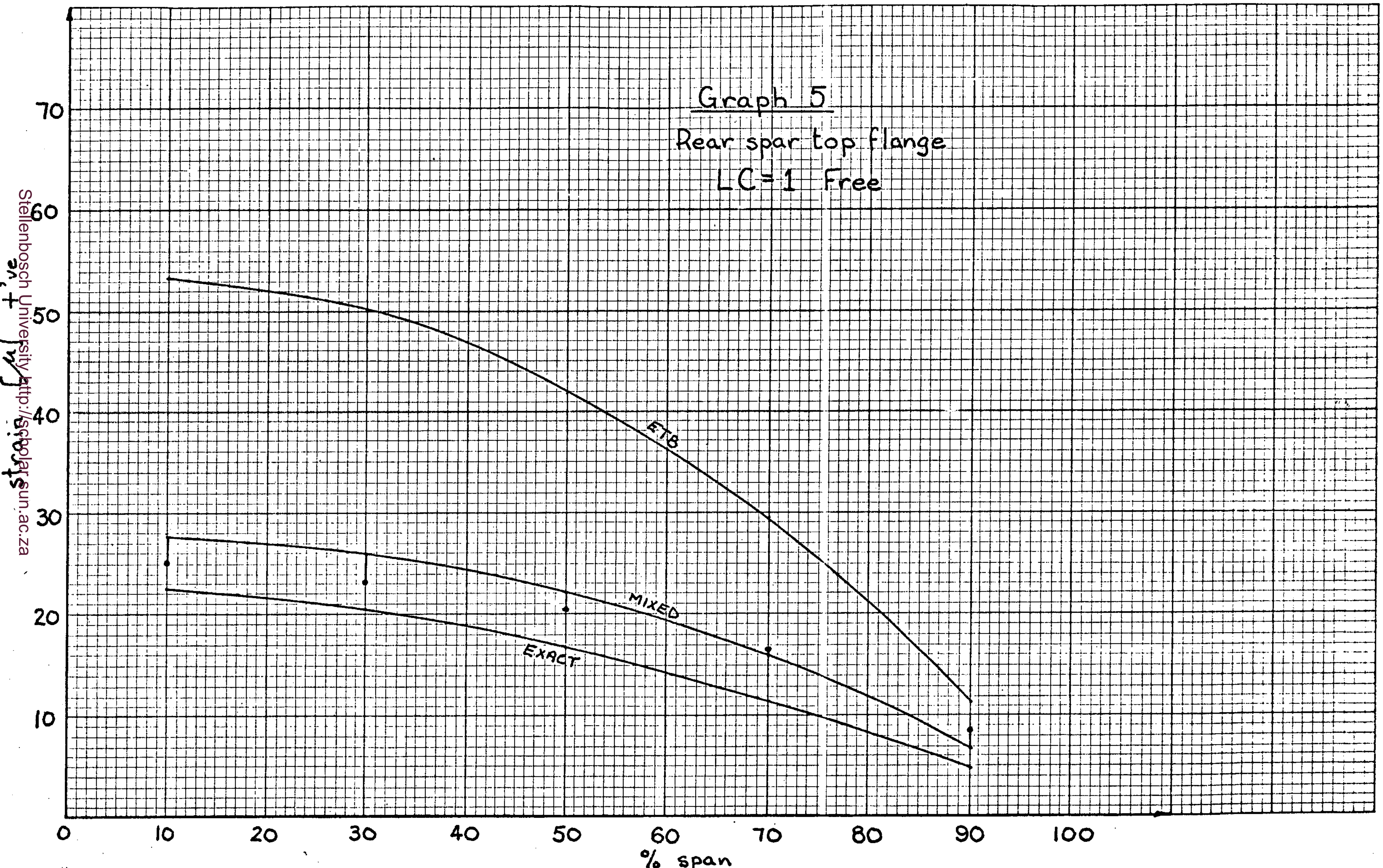

Graph 1
Main spar top flange
LC=1 Free
strain [μ] +'ve
% span
0
10
20
30
40
50
60
70
80
90
100
ETB
MIXED
EXACT

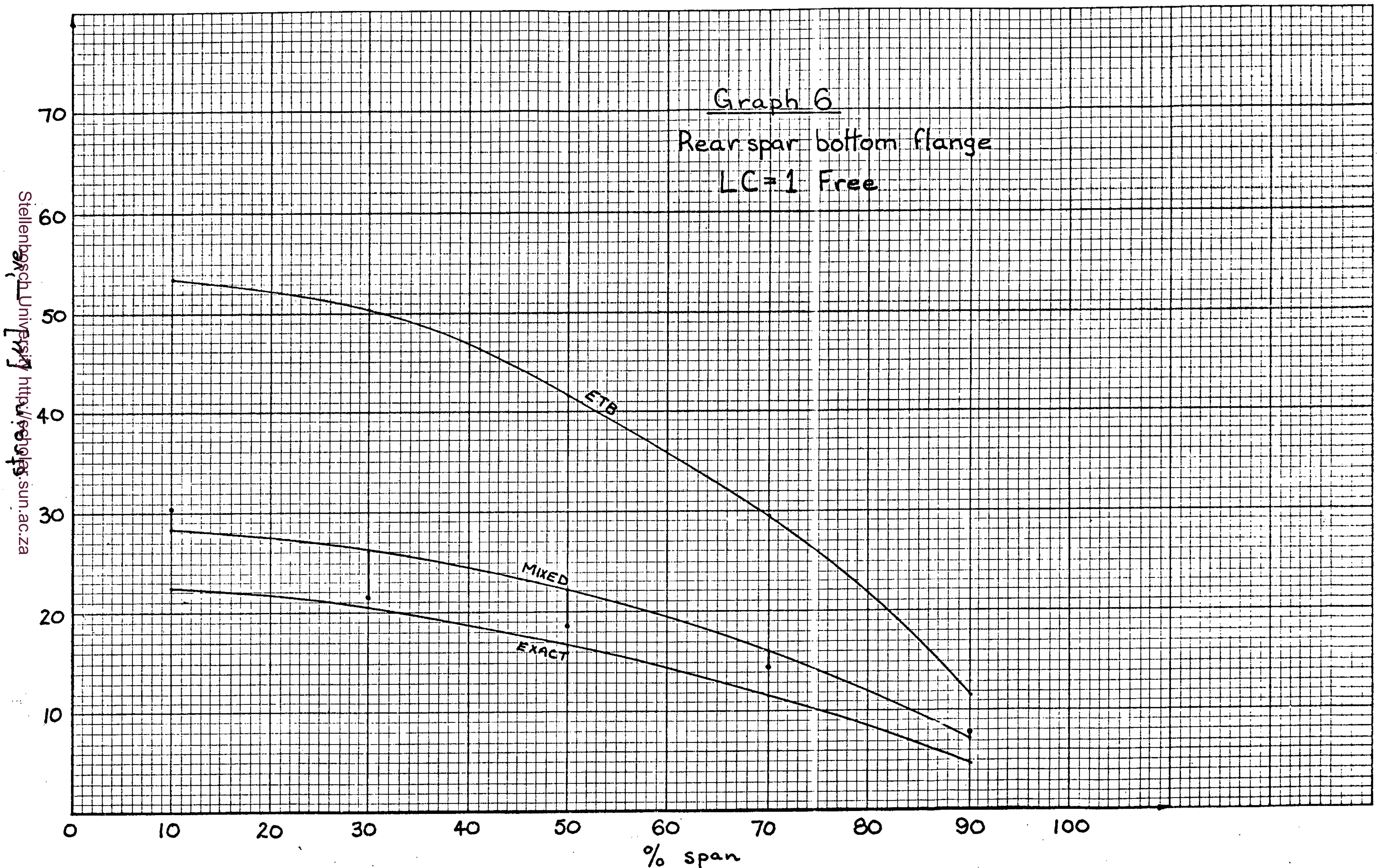

Graph 2
Main spar bottom flange
LC=1 Free
strain [μ] -'ve
% span
0
10
20
30
40
50
60
70
80
90
100
ETB
MIXED
EXACT

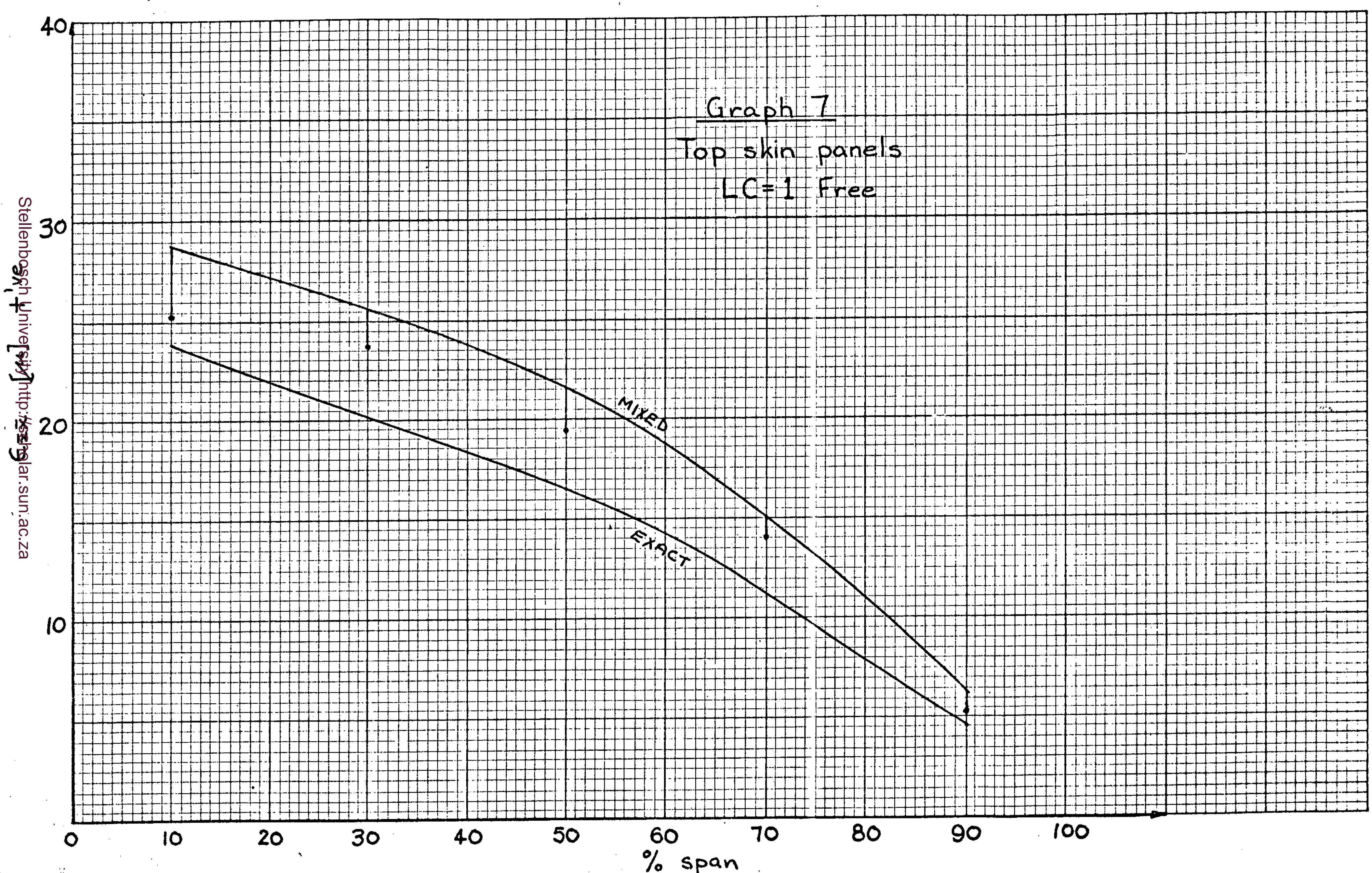

strain [μ] +'ve
% span
EXACT
MIXED
ETB
Graph 3
Top skin stiffener
LC=1 Free

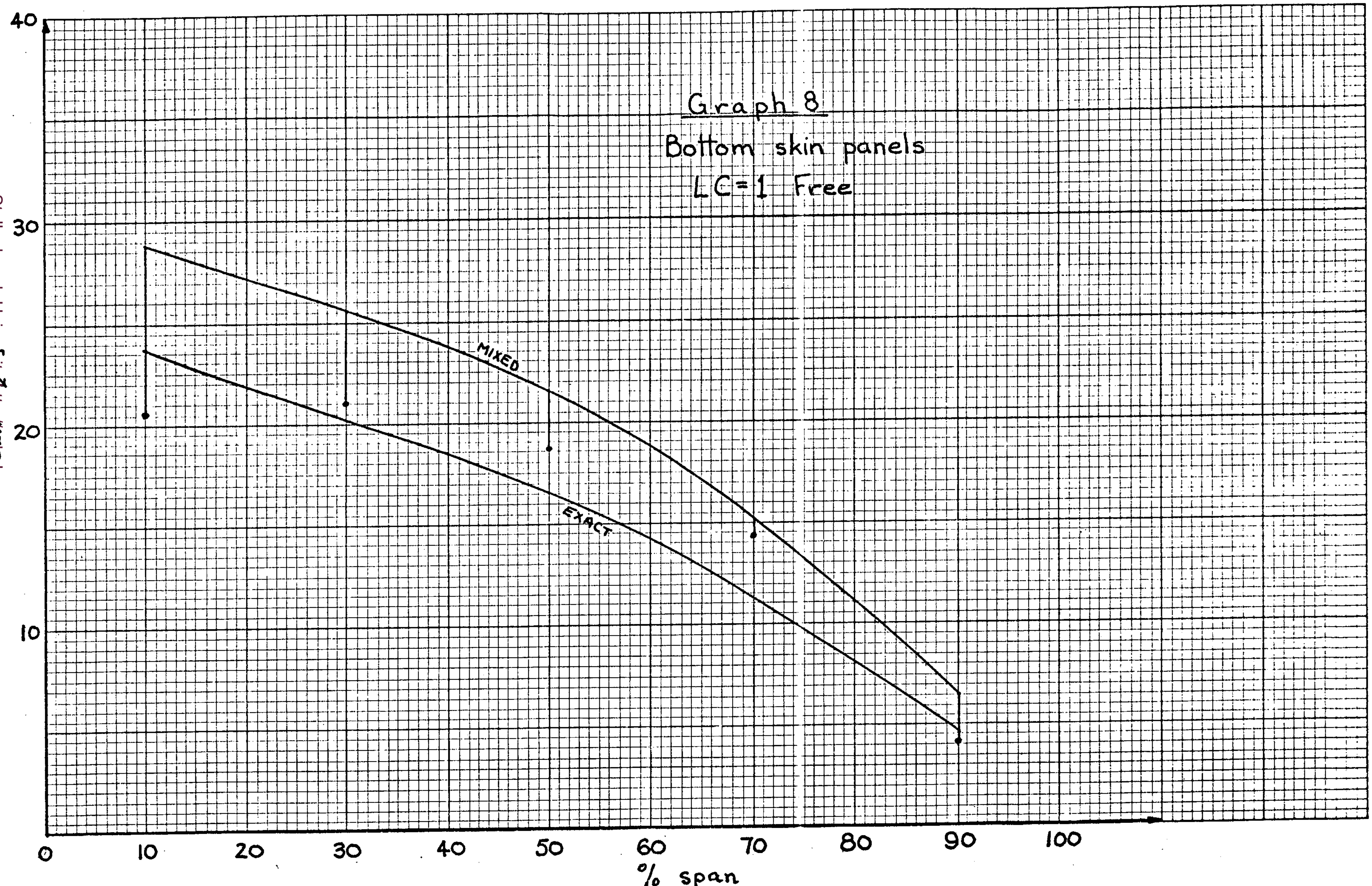


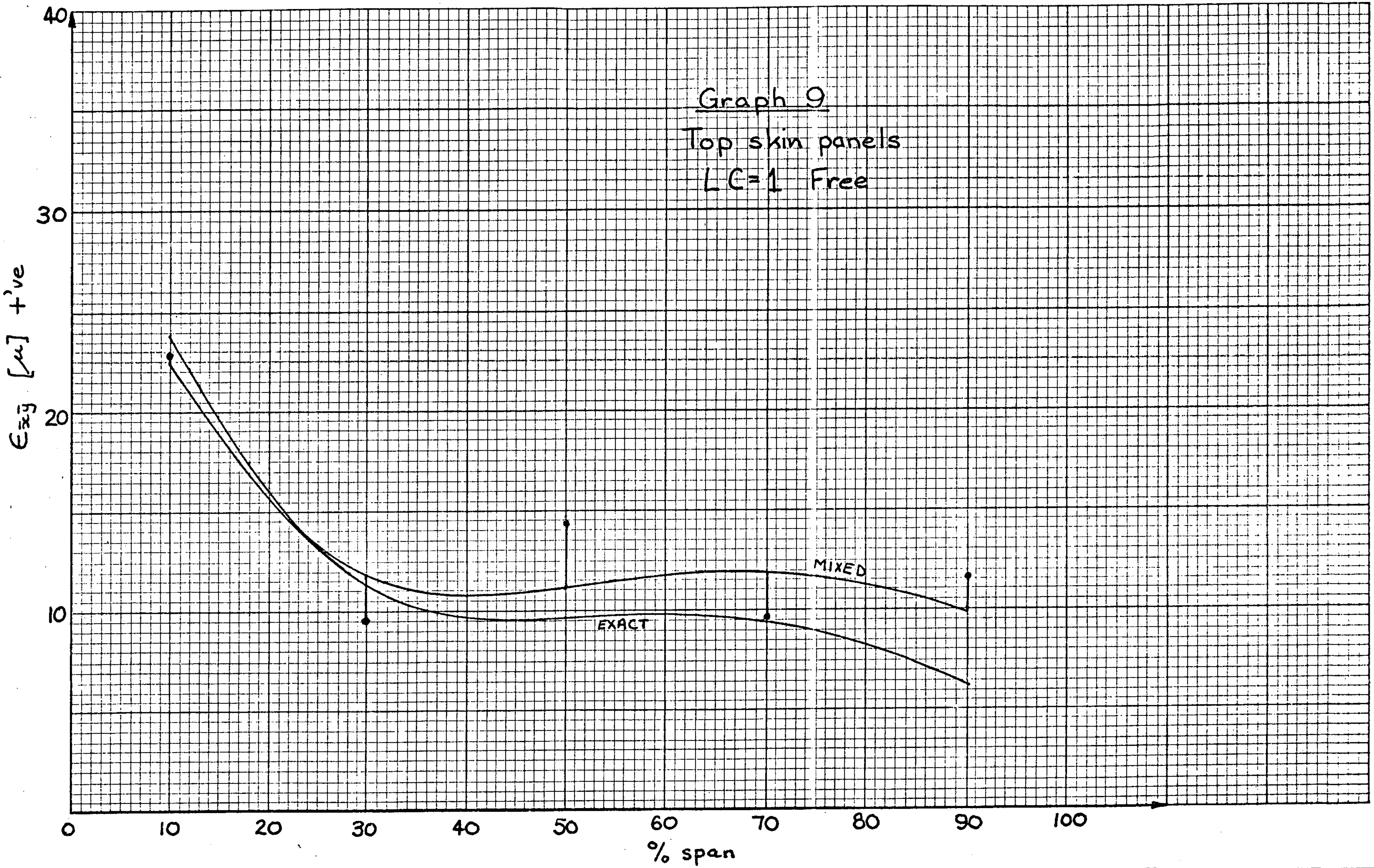
Graph 4
Bottom skin stiffener
LC = 1 Free
ETB
MIXED
EXACT
strain [μ] -'ve
% span
0
10
20
30
40
50
60
70
80
90
100

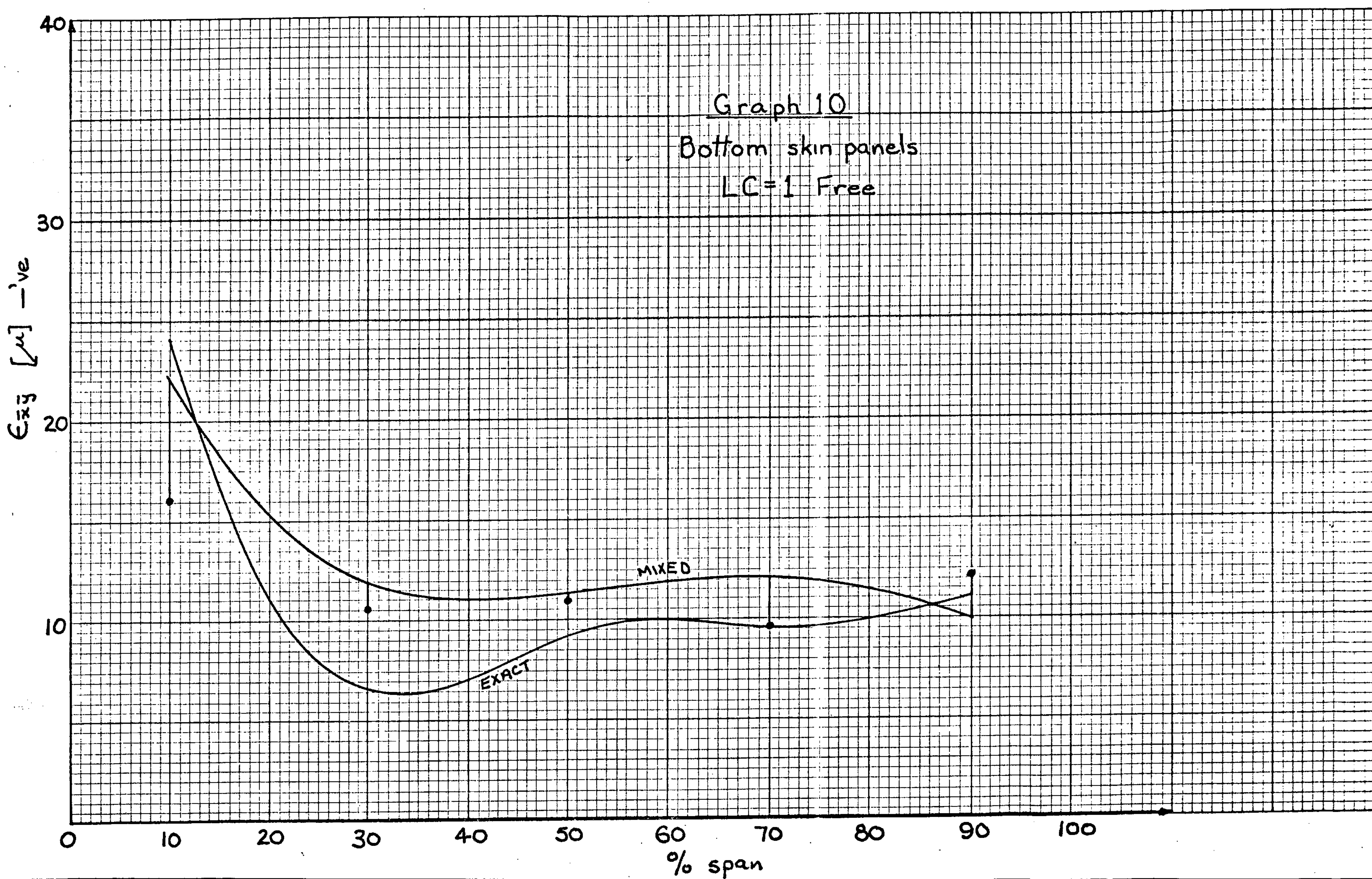

Graph 5
Rear spar top flange
LC=1 Free
ETB
MIXED
EXACT
strain [μ] +ve
% span
0
10
20
30
40
50
60
70
80
90
100
10
20
30
40
50
60
70

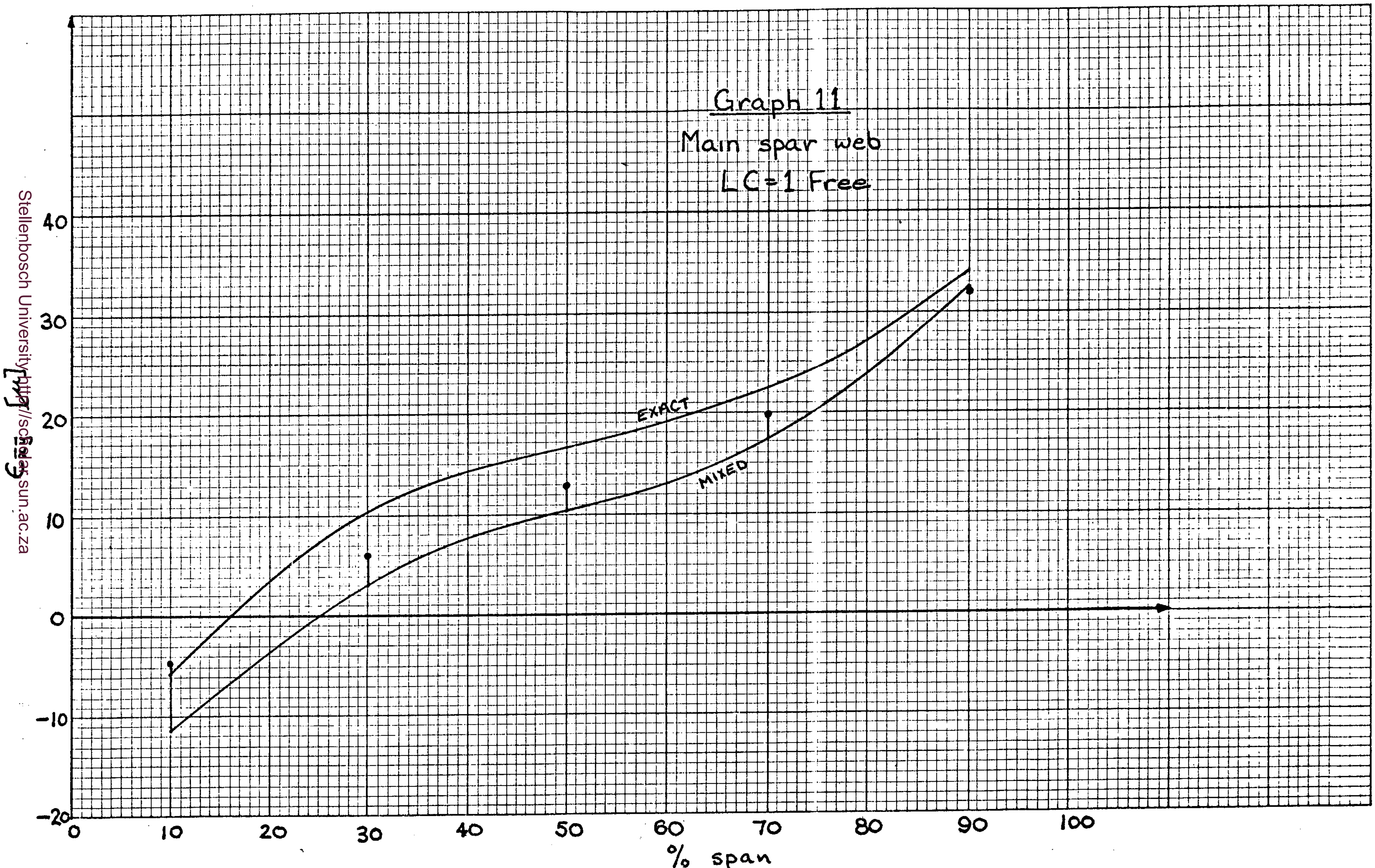

Graph 6
Rear spar bottom flange
LC = 1 Free
ETB
MIXED
EXACT
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
% span
stress
-ve

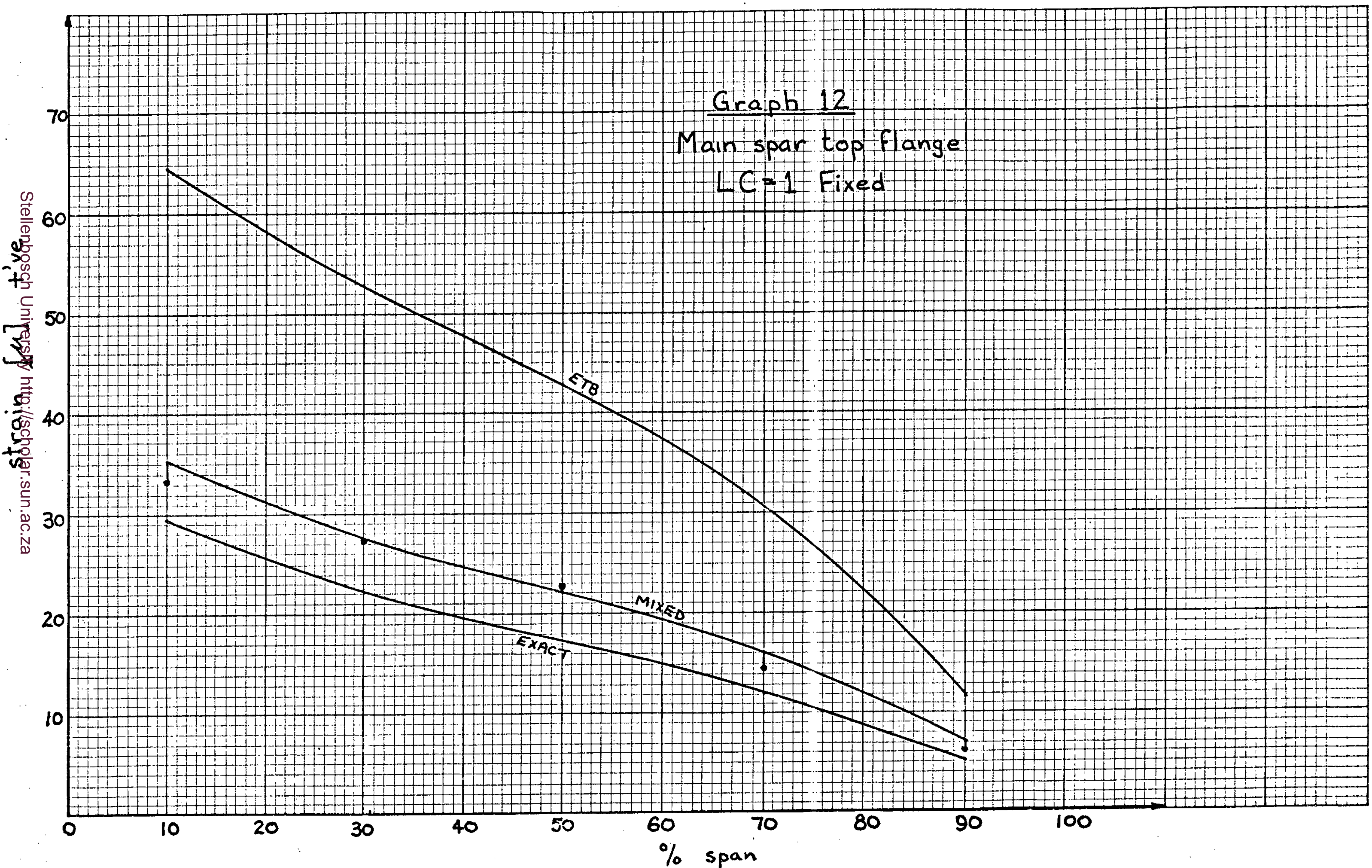

Graph 7
Top skin panels
LC=1 Free
MIXED
EXACT
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
% span

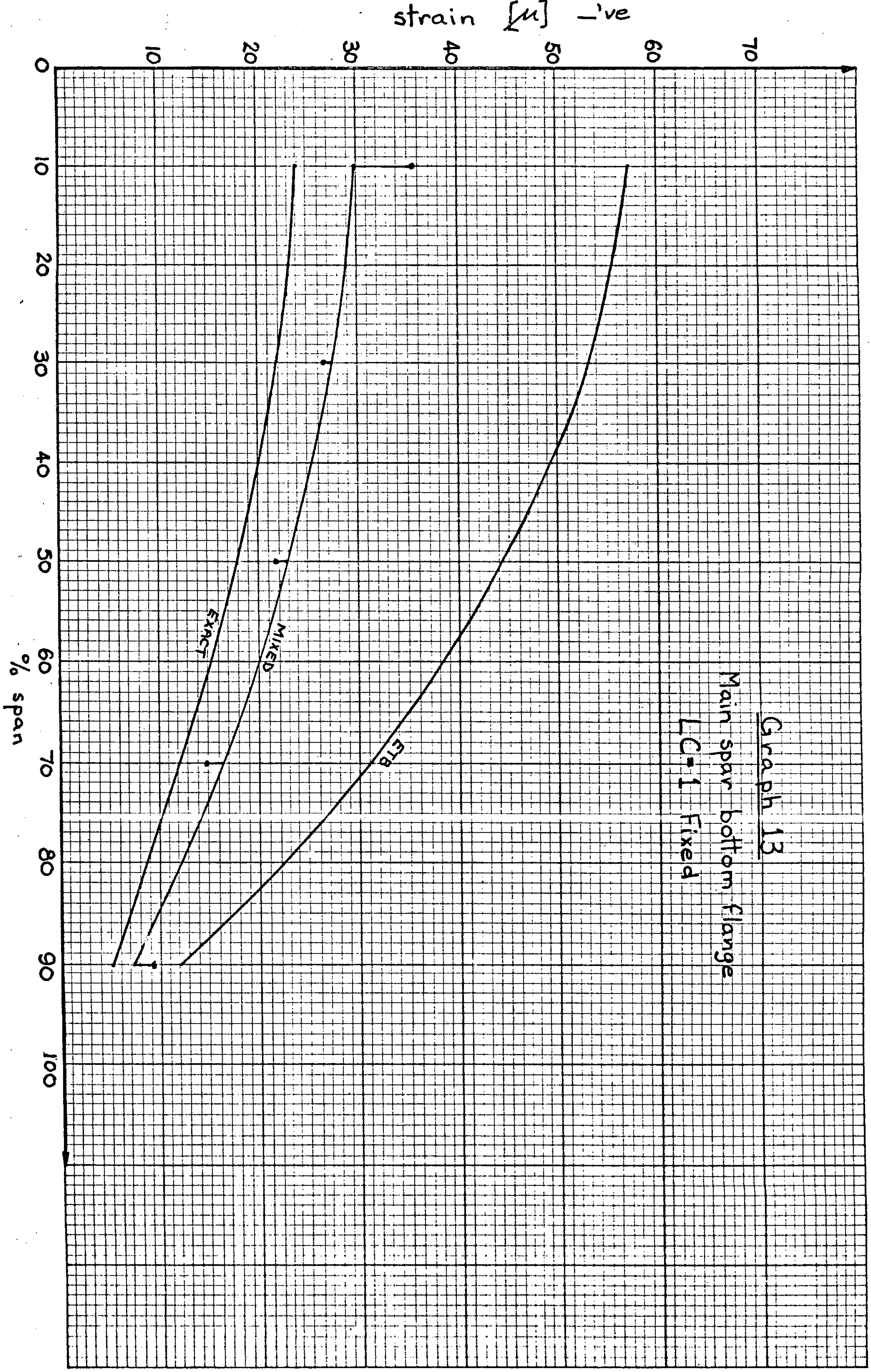

Graph 8
Bottom skin panels
LC=1 Free
MIXED
EXACT
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
% span

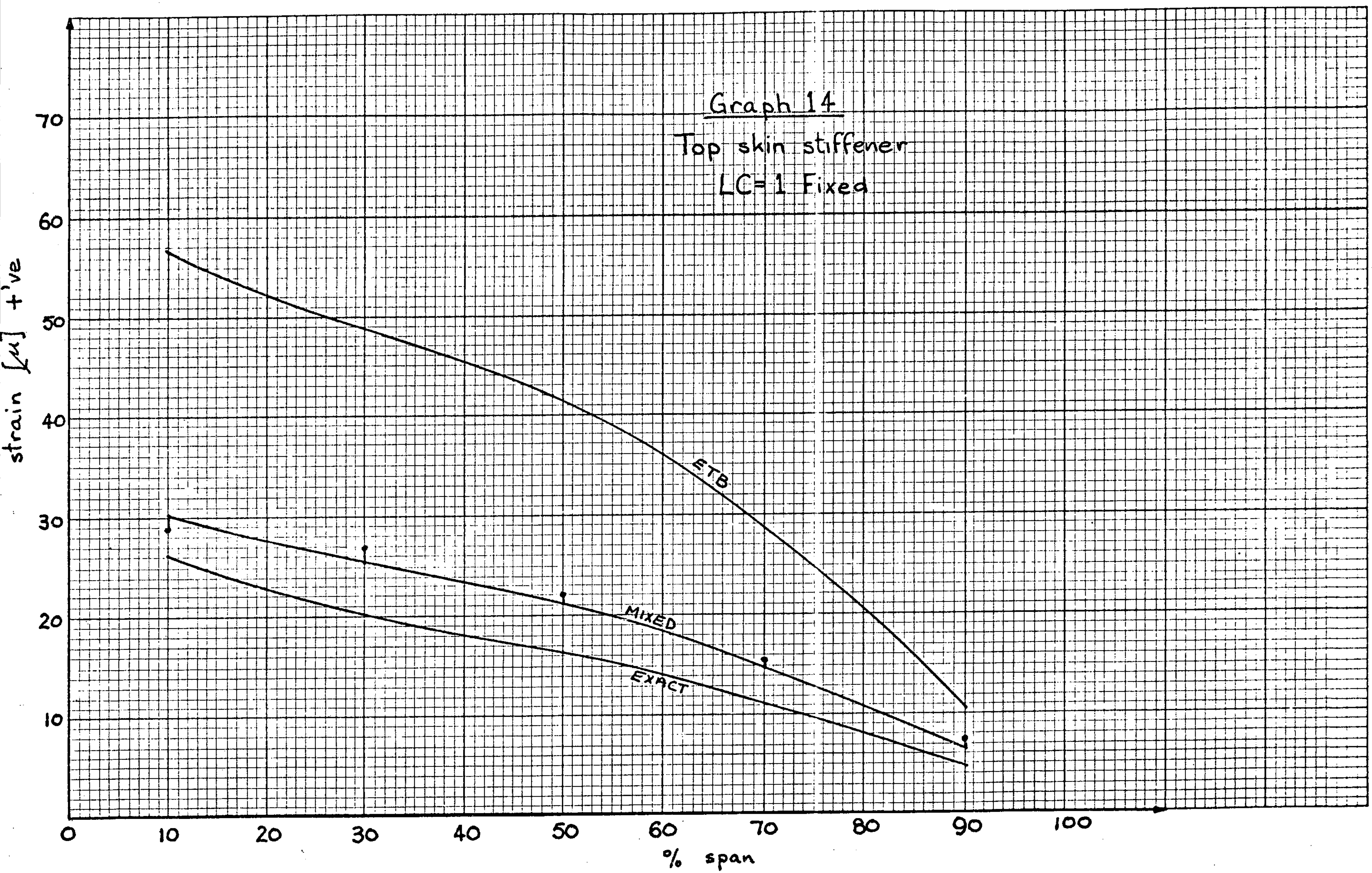

Graph 9
Top skin panels
LC=1 Free
$\epsilon_{\bar{x}\bar{y}}$ [μ] +'ve
40
30
20
10
MIXED
EXACT
0
10
20
30
40
50
60
70
80
90
100
% span

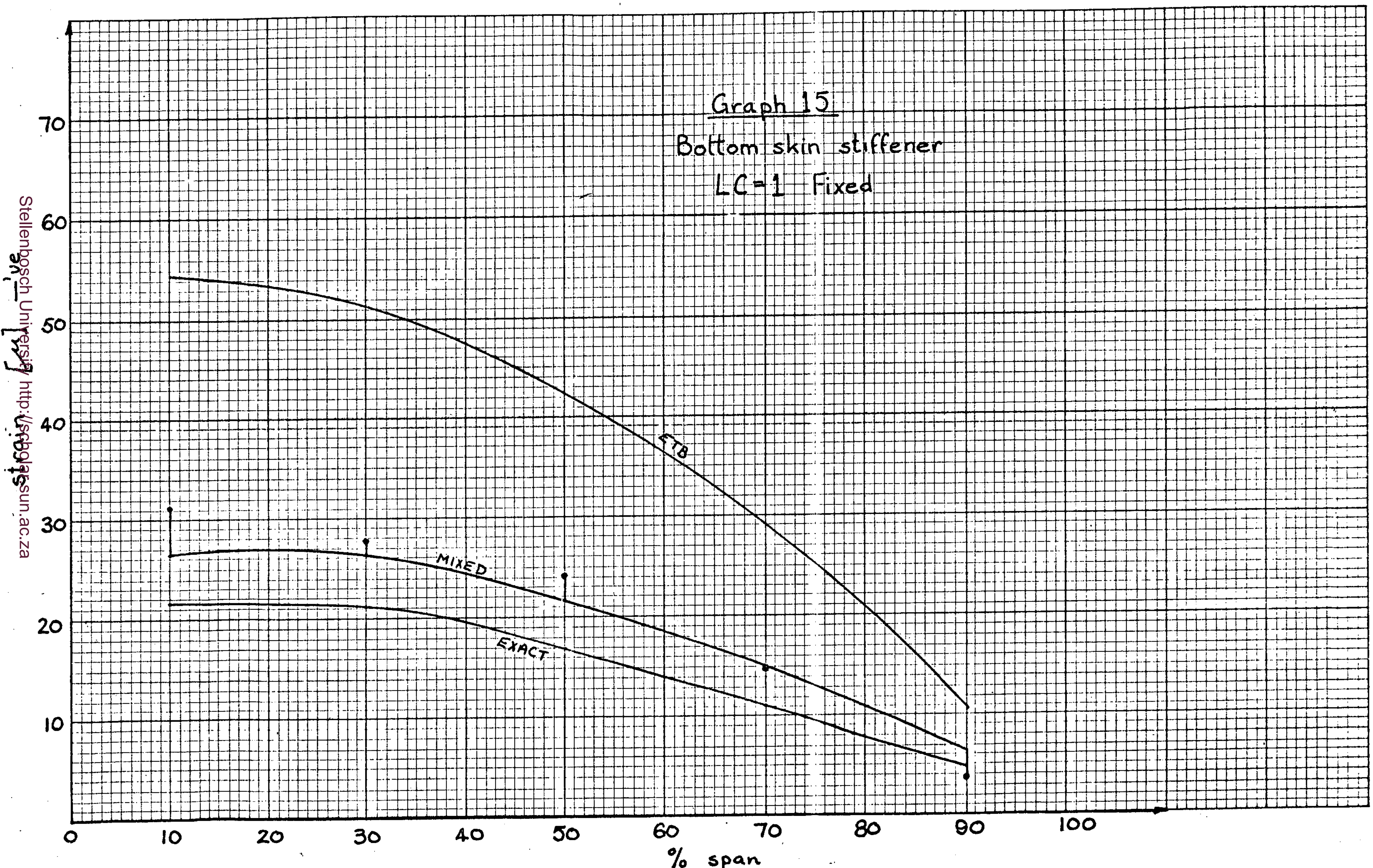

Graph 10
Bottom skin panels
LC = 1 Free
$\epsilon_{\bar{x}\bar{y}}$ [μ] –'ve
% span
MIXED
EXACT
0
10
20
30
40
50
60
70
80
90
100

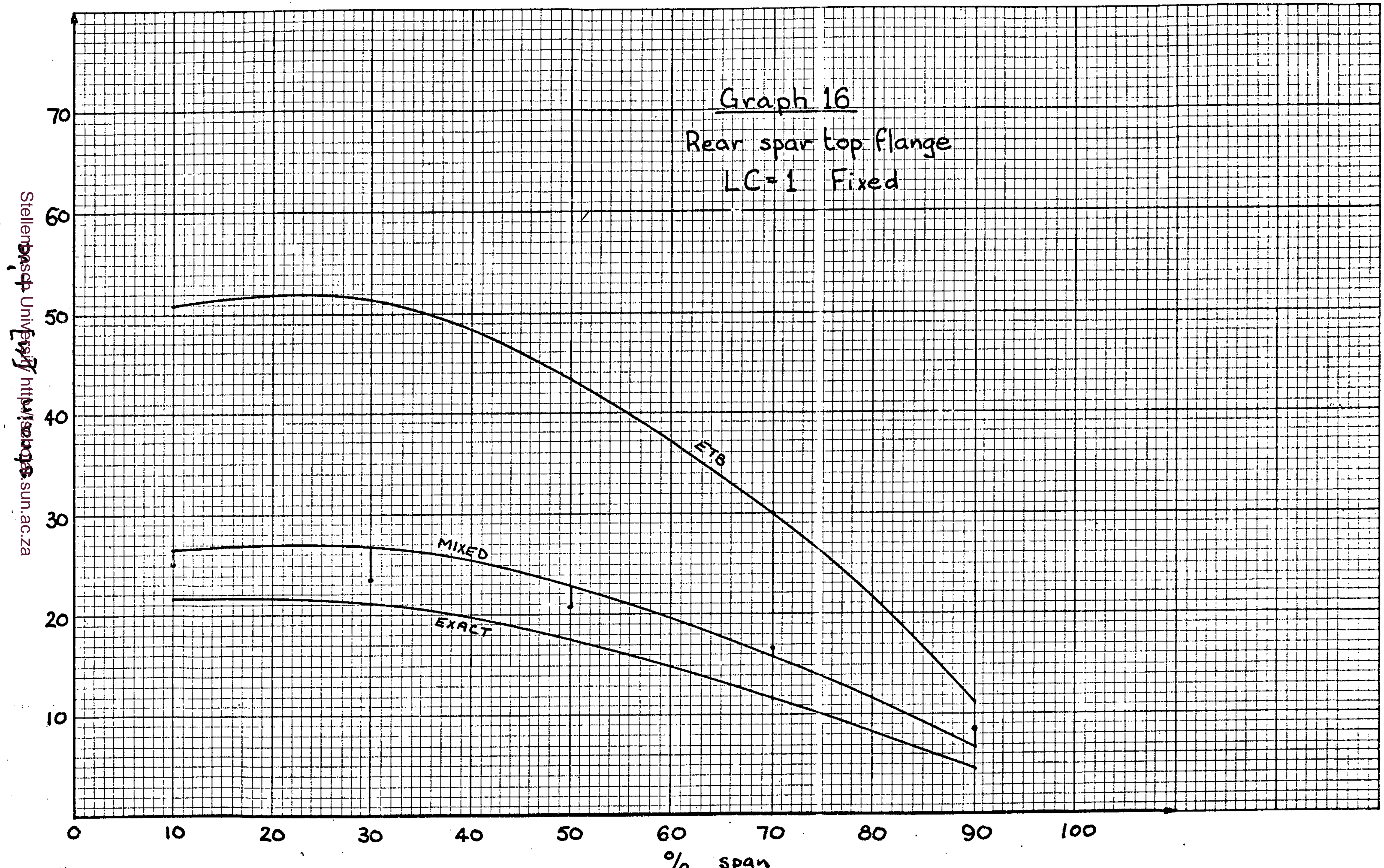

Graph 11
Main spar web
LC=1 Free
EXACT
MIXED
40
30
20
10
0
-10
-20
0
10
20
30
40
50
60
70
80
90
100
% span

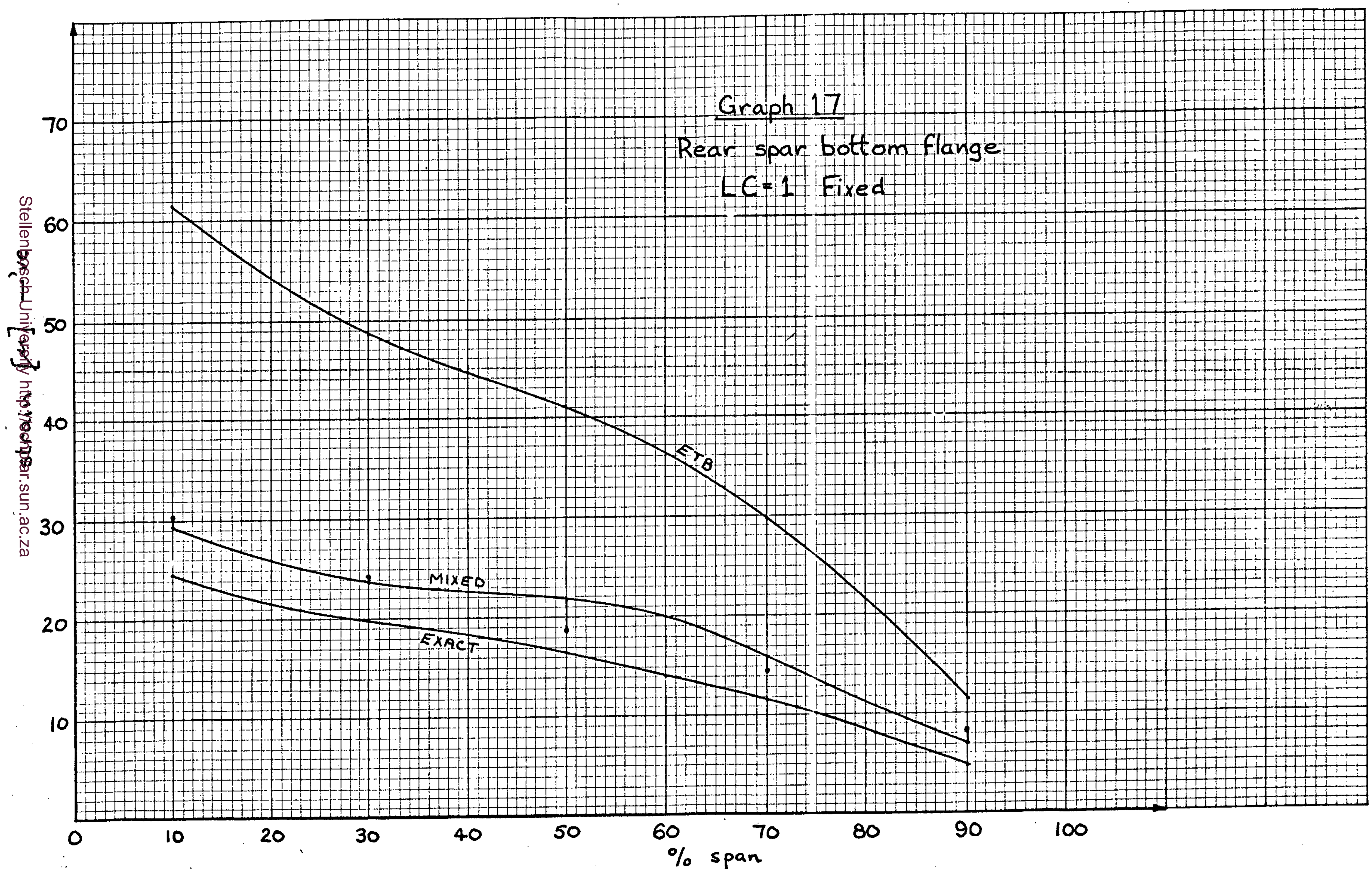

Graph 12
Main spar top flange
LC = 1 Fixed
ETB
MIXED
EXACT
strain [μ] +'ve
% span
0
10
20
30
40
50
60
70
80
90
100

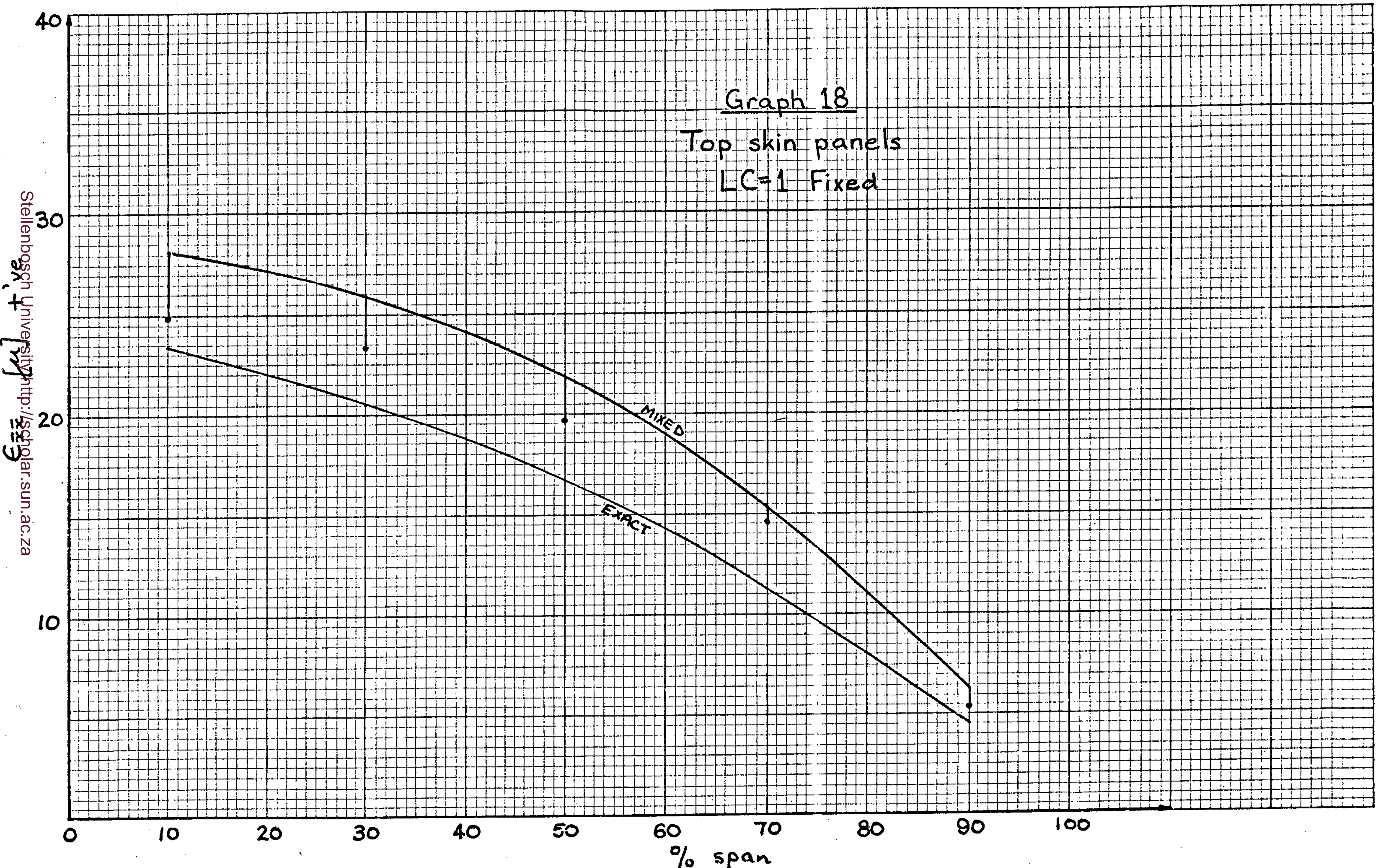

strain [μ] -'ve
0
10
20
30
40
50
60
70
80
90
100
% span
EXACT
MIXED
ETB
Graph 13
Main spar bottom flange
LC-1 Fixed


Graph 14
Top skin stiffener
LC=1 Fixed
strain [μ] +'ve
% span
ETB
MIXED
EXACT
0
10
20
30
40
50
60
70
80
90
100

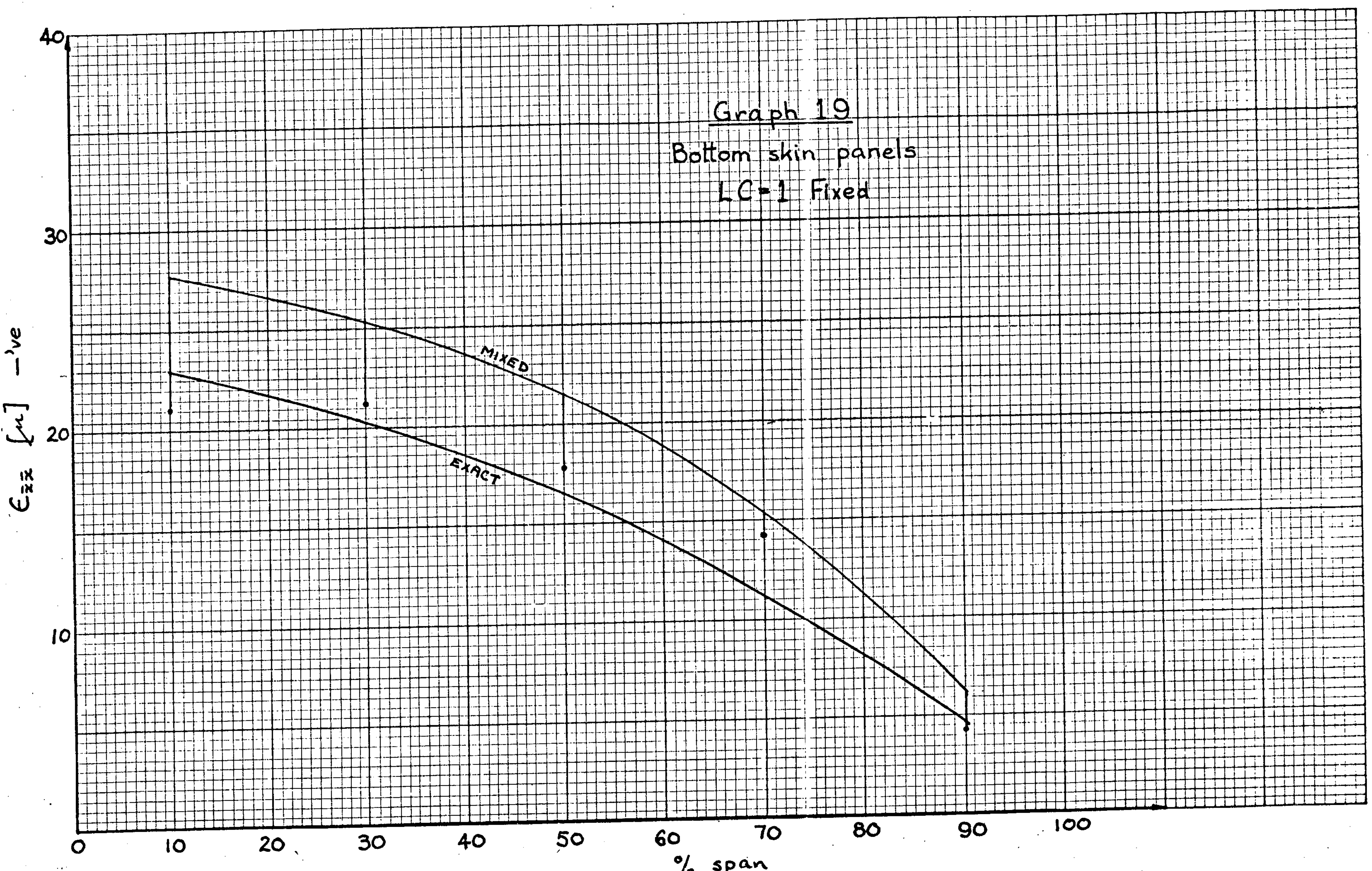

Graph 15
Bottom skin stiffener
LC=1 Fixed
ETB
MIXED
EXACT
strain [μ] -ve
% span
0
10
20
30
40
50
60
70
80
90
100


Graph 16
Rear spar top flange
LC=1 Fixed
ETB
MIXED
EXACT
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
% span

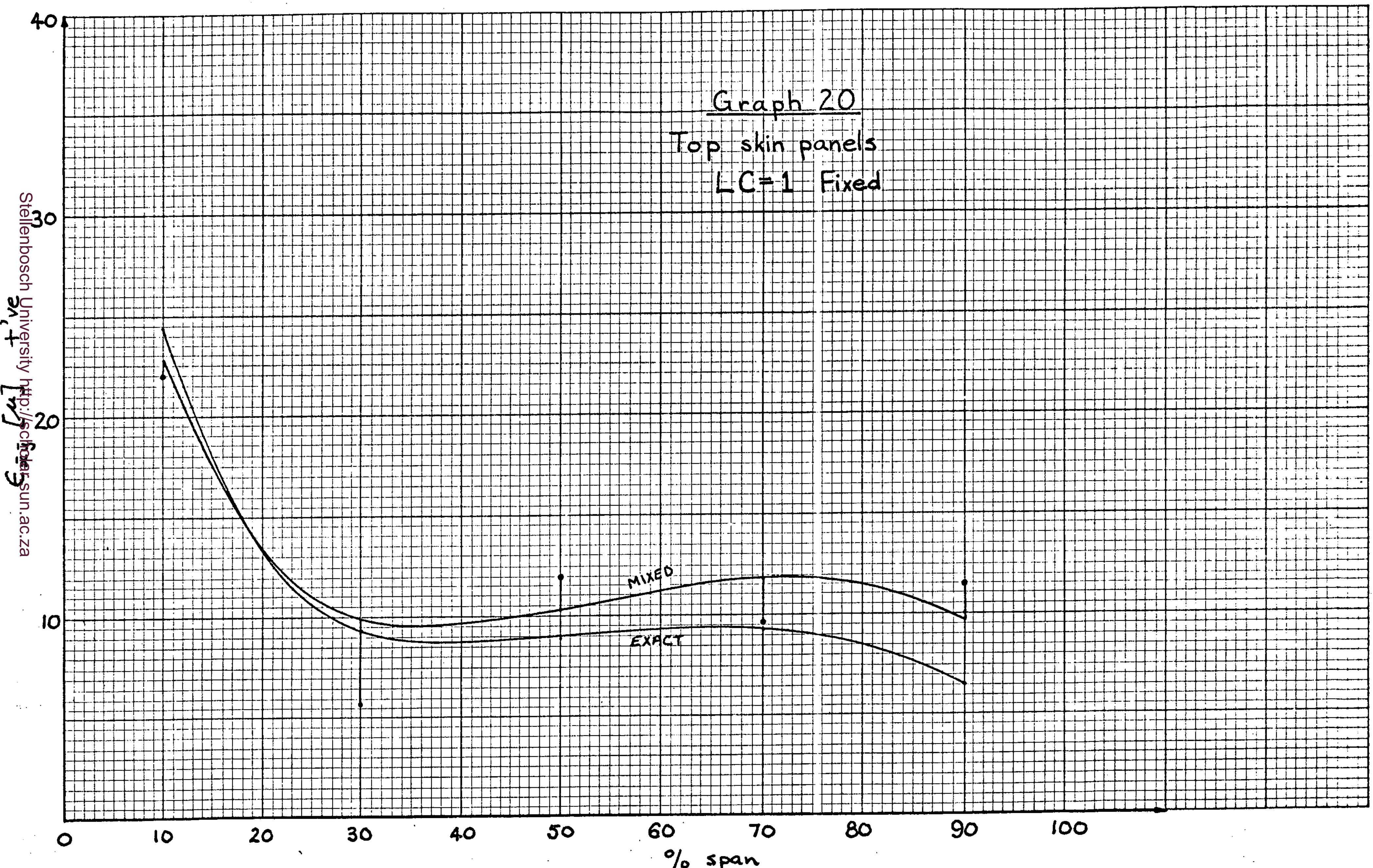

Graph 17
Rear spar bottom flange
LC=1 Fixed
ETB
MIXED
EXACT
% span
0
10
20
30
40
50
60
70
80
90
100

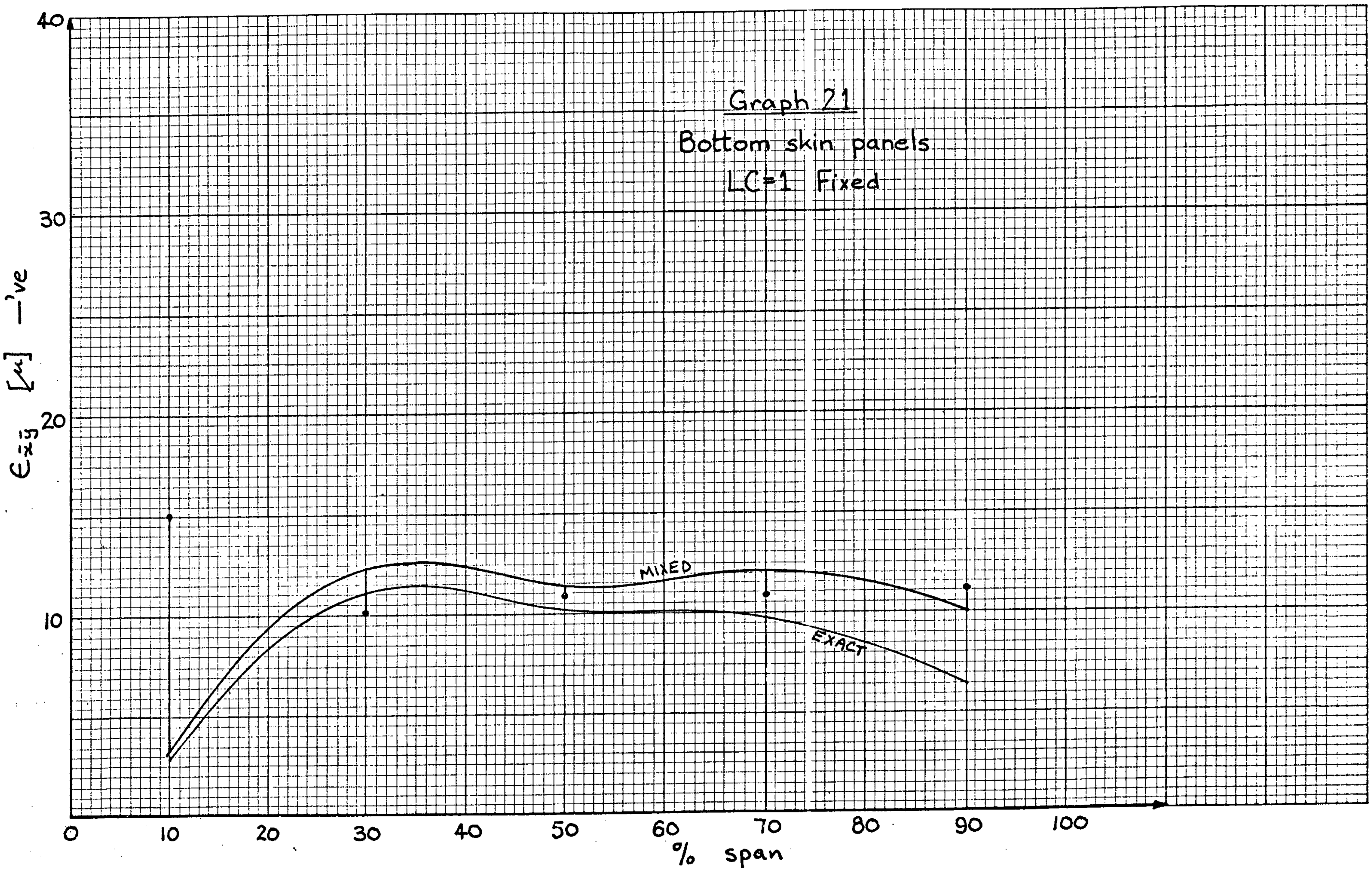


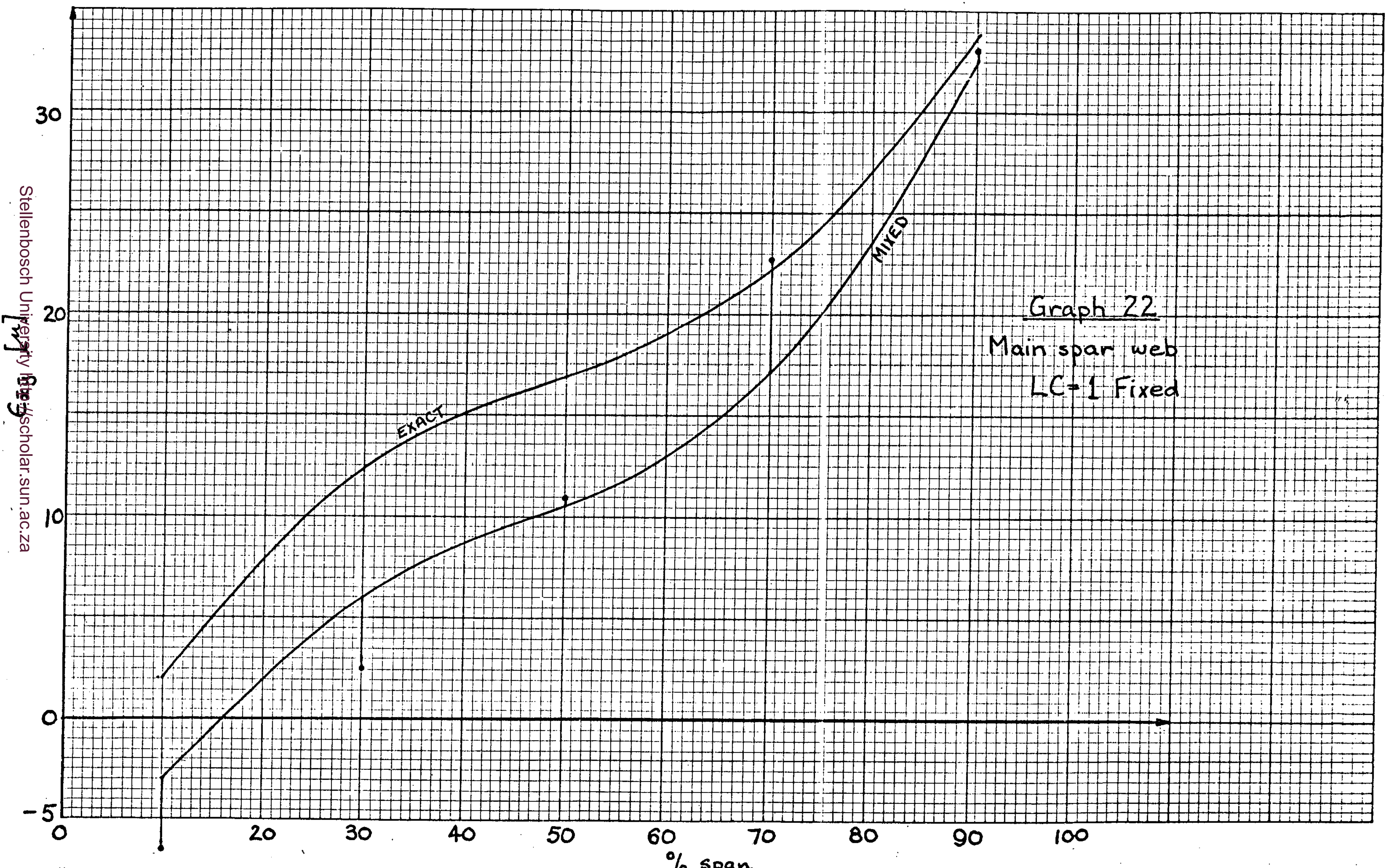
Graph 18
Top skin panels
LC=1 Fixed
MIXED
EXACT
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
% span

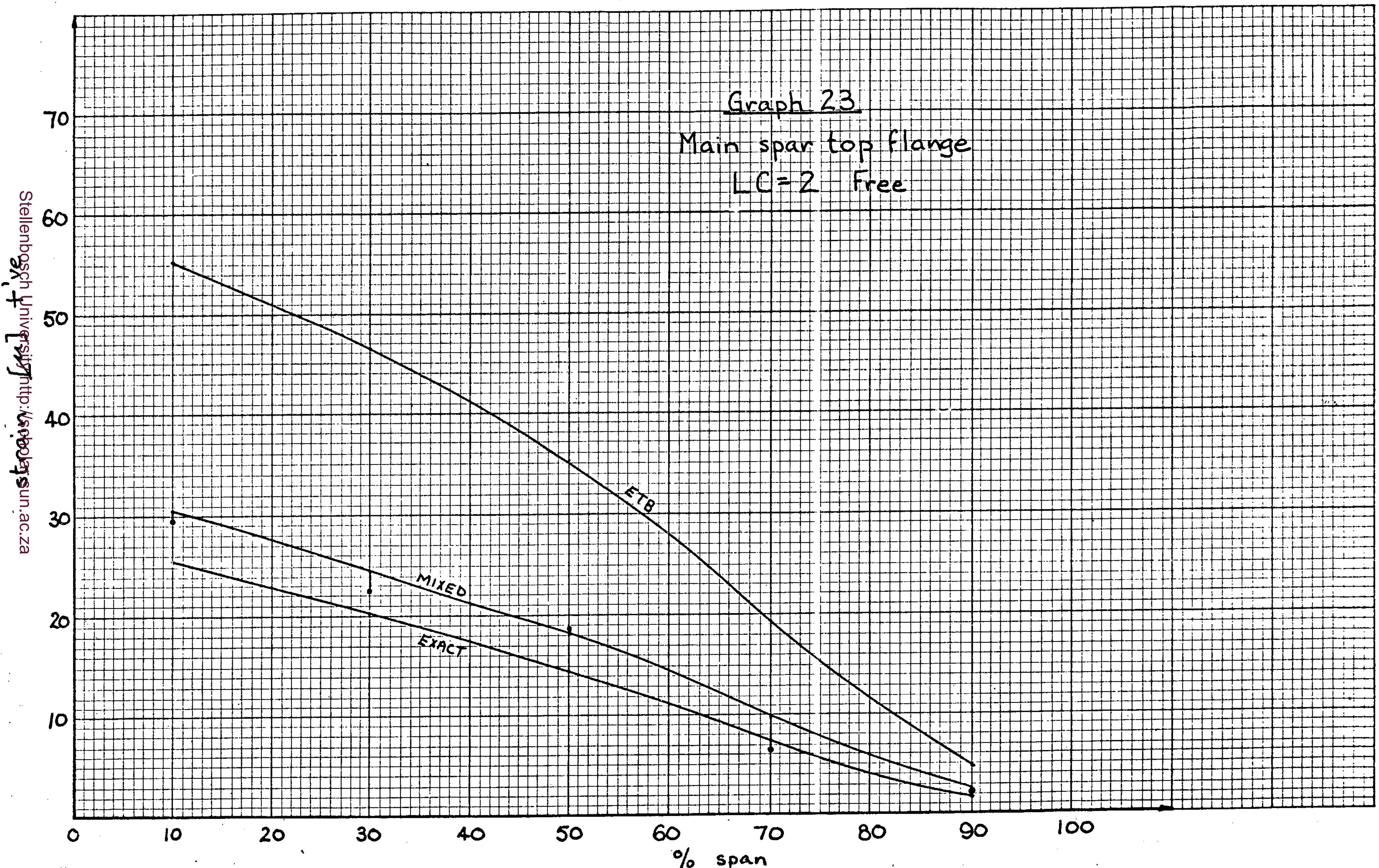


Graph 19
Bottom skin panels
LC=1 Fixed

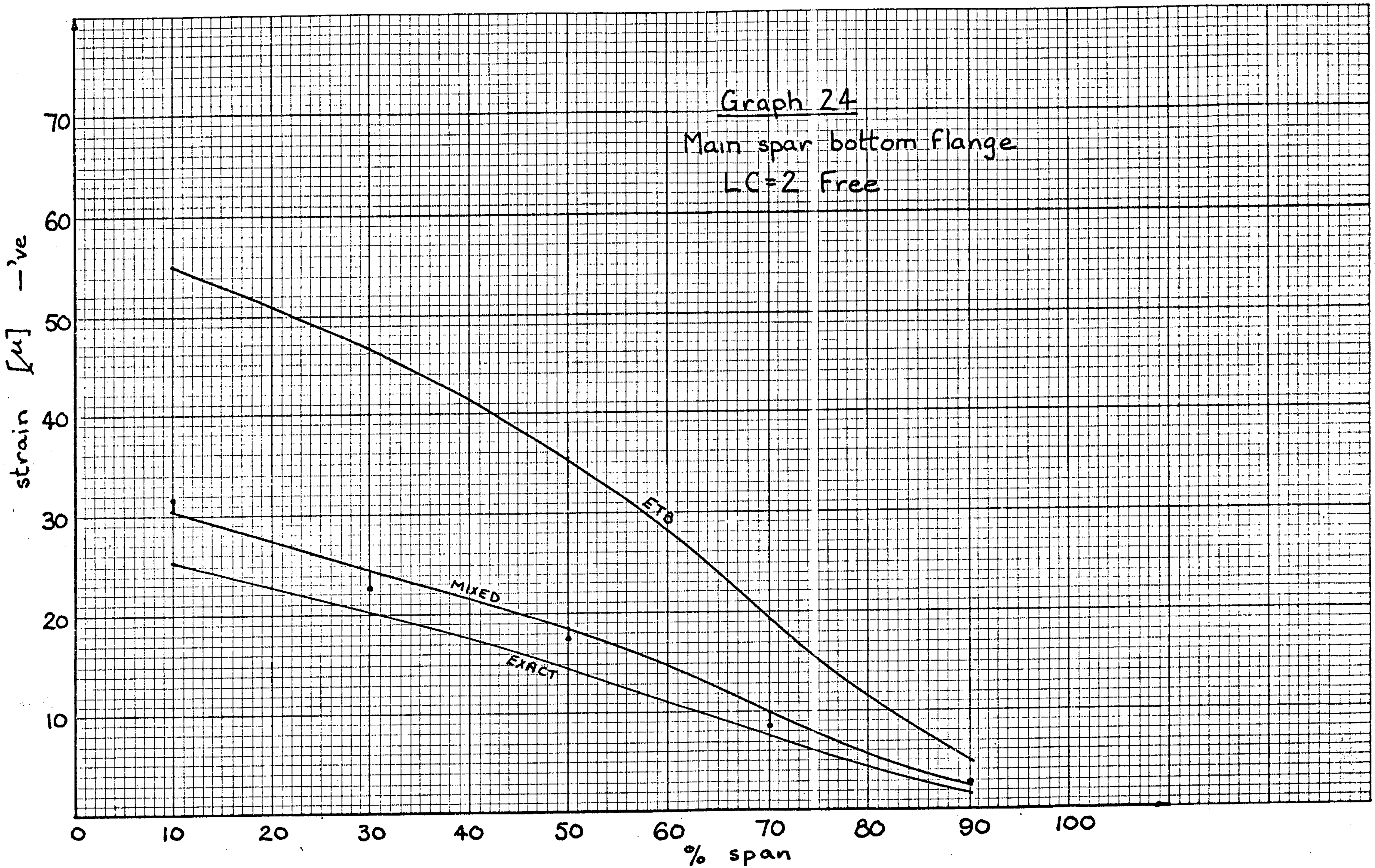

Graph 20
Top skin panels
LC=1 Fixed
MIXED
EXACT
% span
+'ve
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100


Graph 21
Bottom skin panels
LC=1 Fixed
MIXED
EXACT
$\epsilon_{\bar{x}\bar{y}}$ [μ] –'ve
0
10
20
30
40
50
60
70
80
90
100
% span

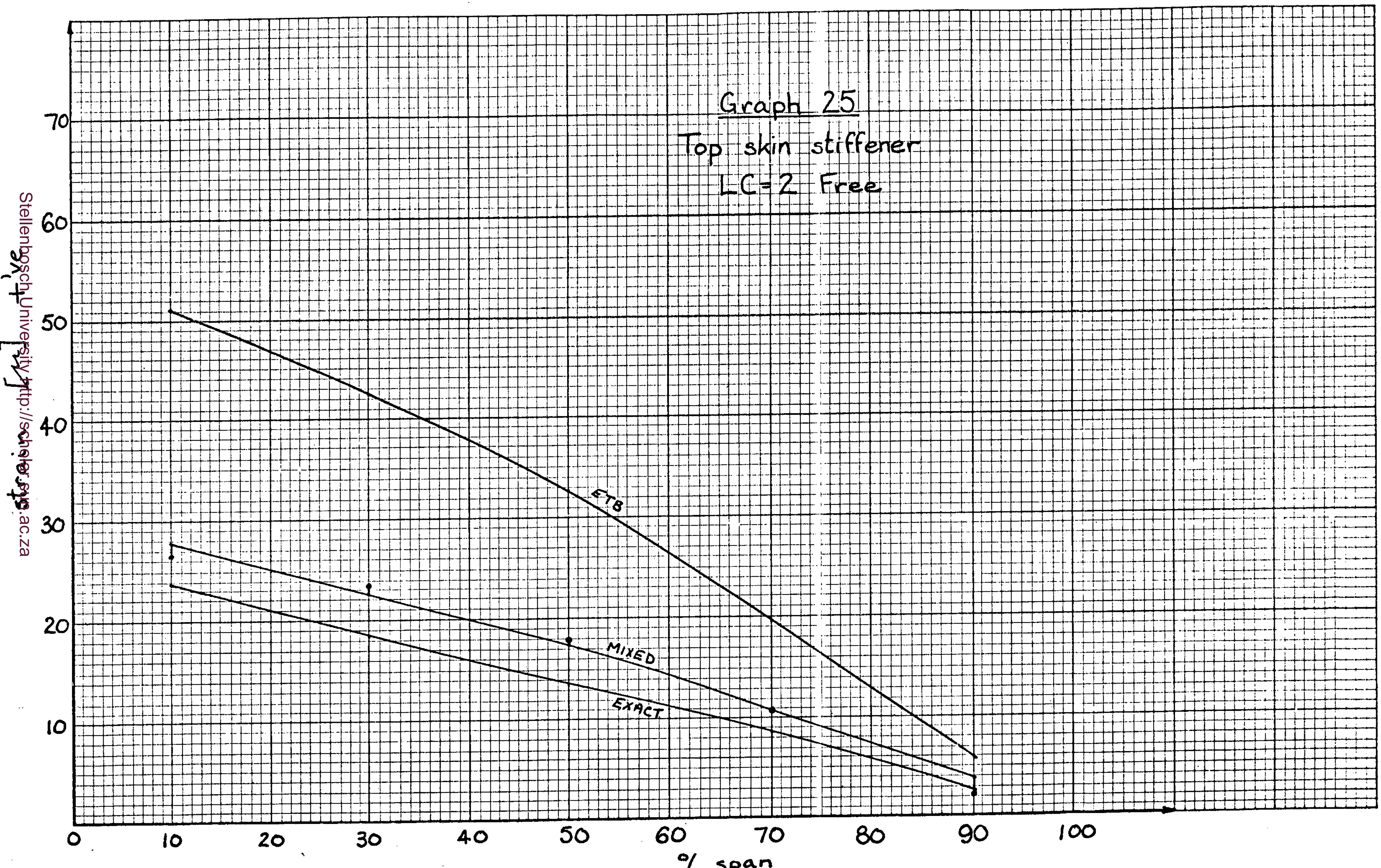

Graph 22
Main spar web
LC=1 Fixed
EXACT
MIXED
30
20
10
0
-5
0
20
30
40
50
60
70
80
90
100
% span



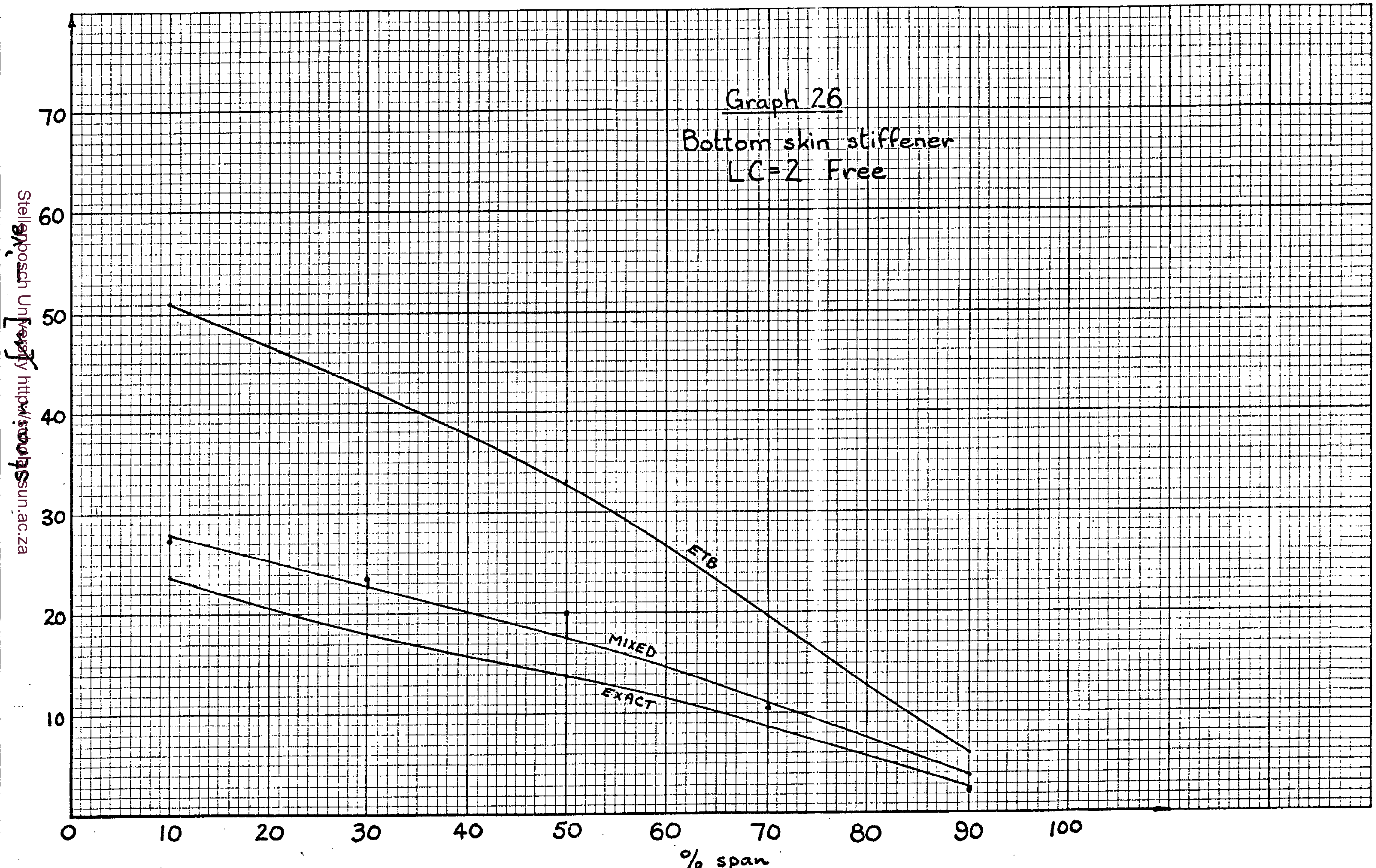
Graph 23
Main spar top flange
LC=2 Free
ETB
MIXED
EXACT
strain [με] +'ve
% span
0
10
20
30
40
50
60
70
80
90
100



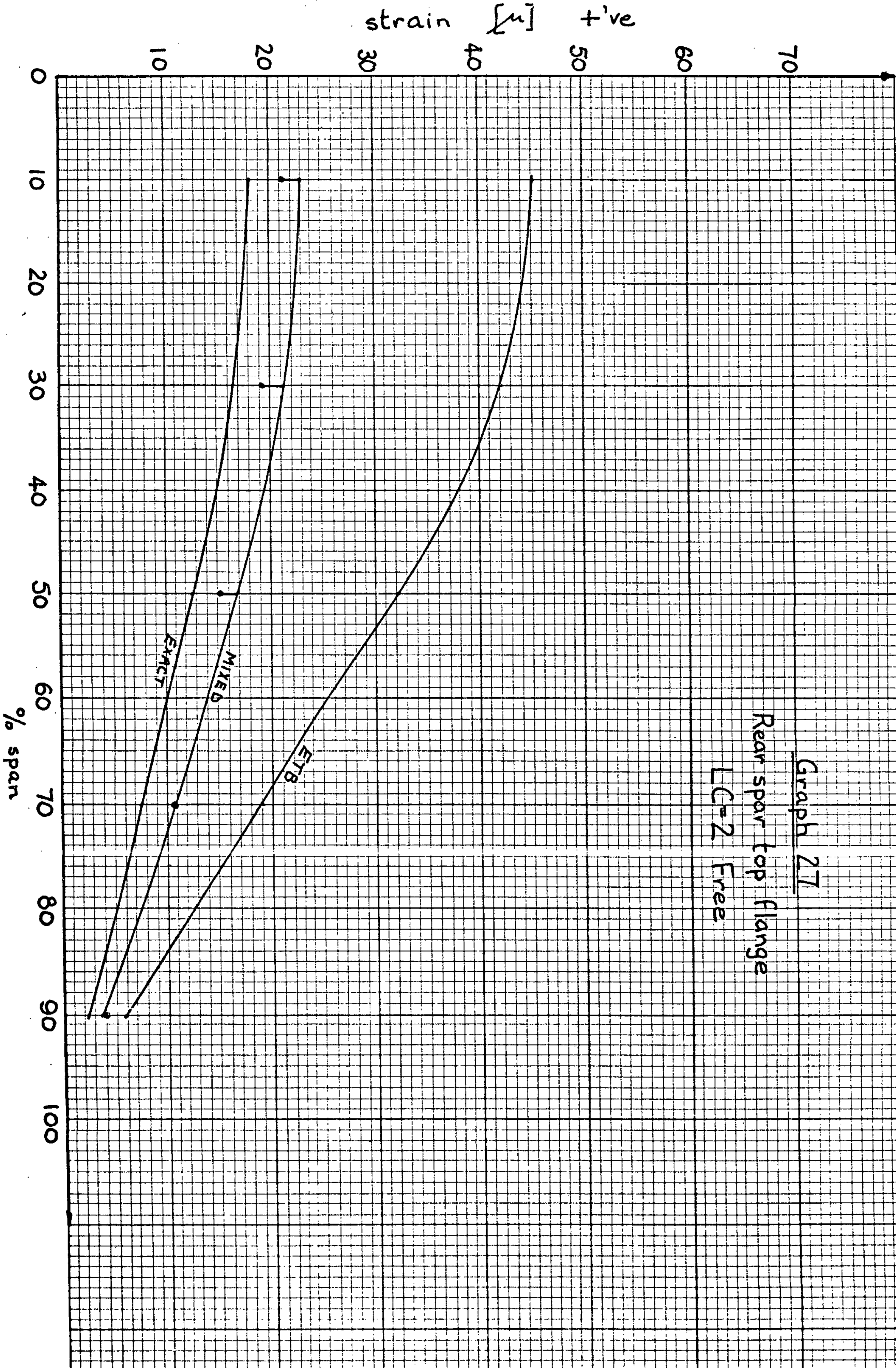
Graph 24
Main spar bottom flange
LC=2 Free
strain [μ] —'ve
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
% span
ETB
MIXED
EXACT

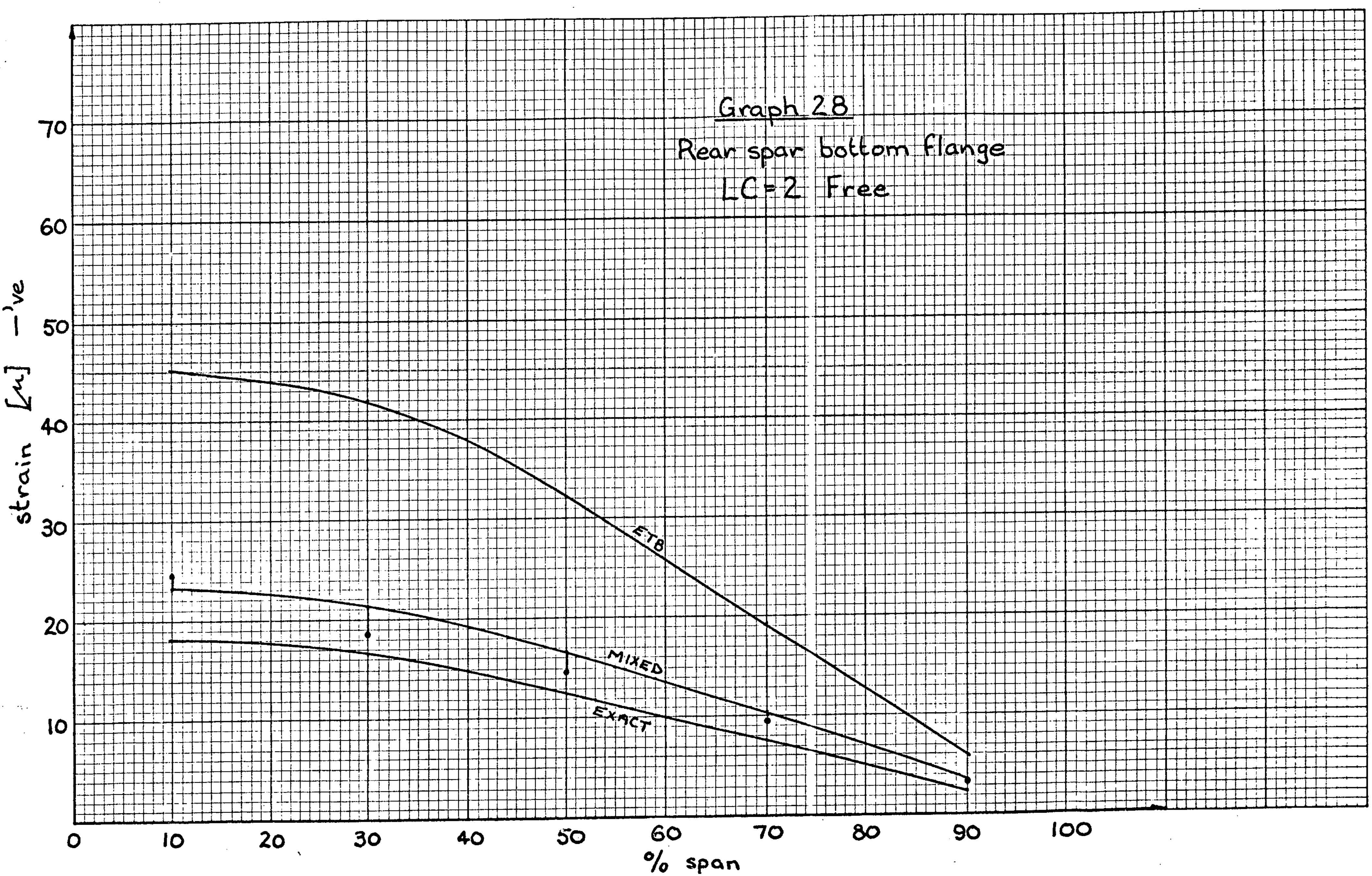


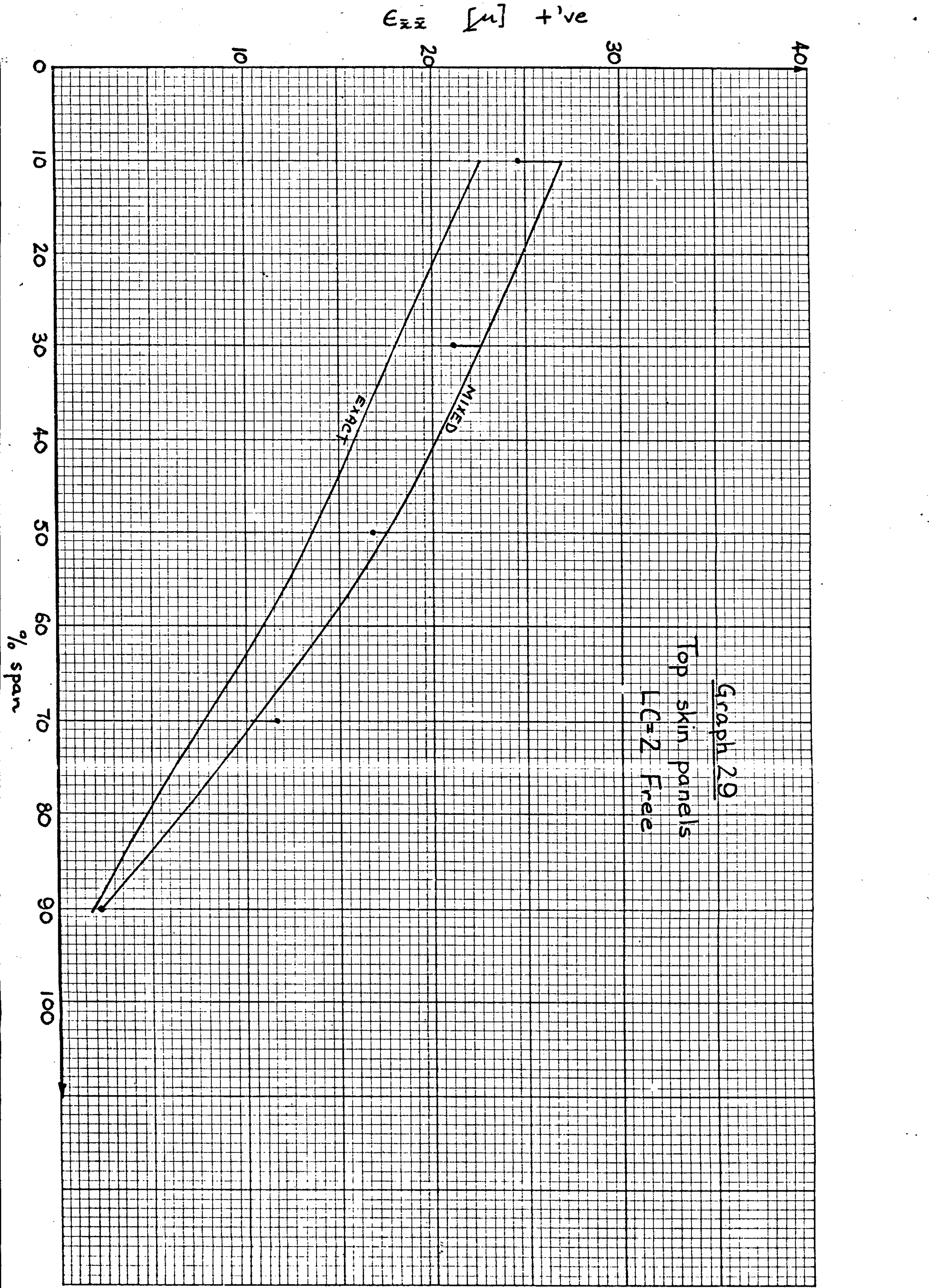
Graph 25
Top skin stiffener
LC=2 Free
ETB
MIXED
EXACT
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
% span
strain [με] +ve

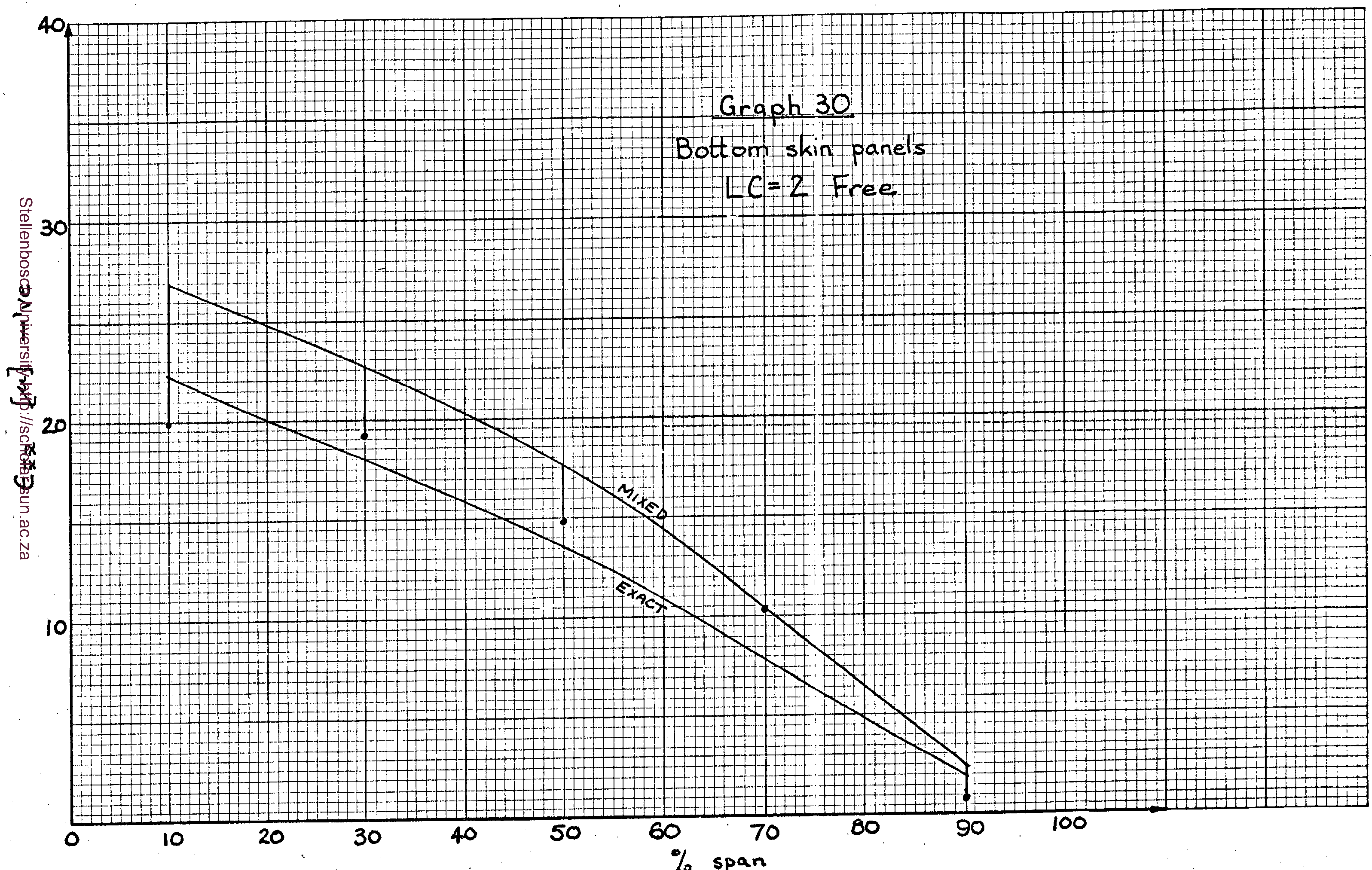


Graph 26
Bottom skin stiffener
LC=2 Free
ETB
MIXED
EXACT
0
10
20
30
40
50
60
70
80
90
100
% span

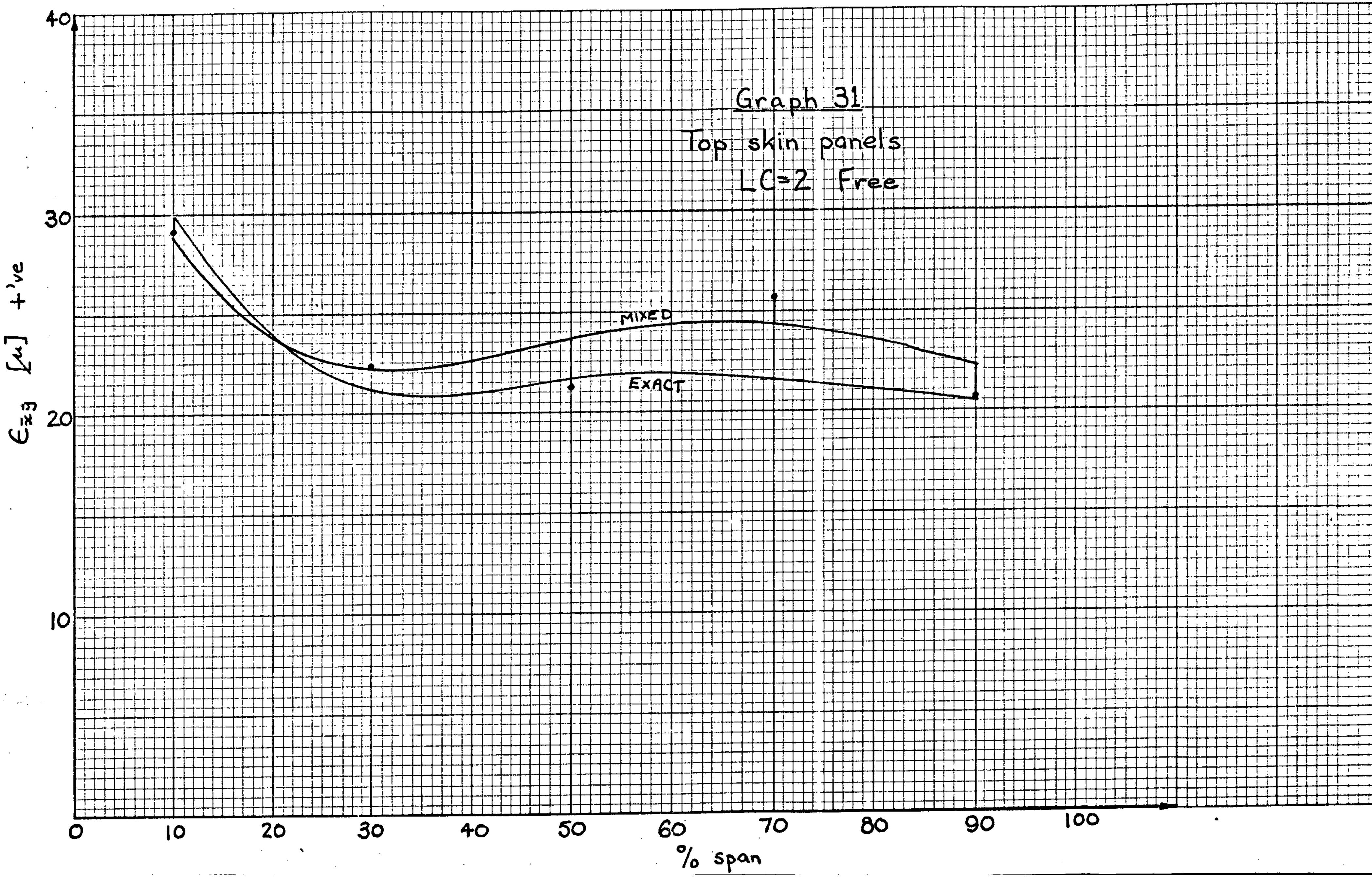

strain [μ] +'ve
0
10
20
30
40
50
60
70
% span
100
EXACT
MIXED
ETB
Graph 27
Rear spar top flange
LC=2 Free

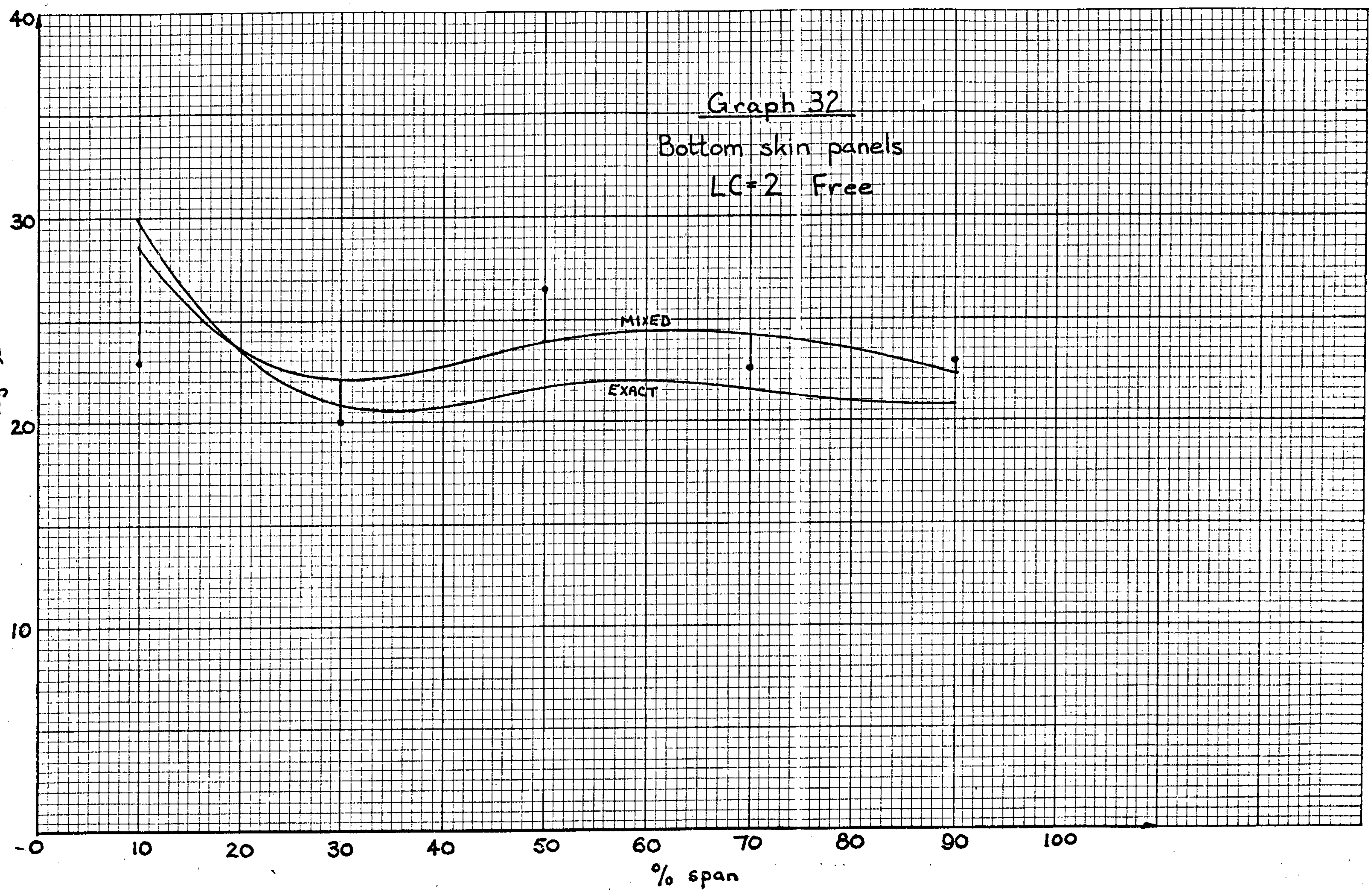

Graph 2.8
Rear spar bottom flange
LC=2 Free
strain [μ] -'ve
0
10
20
30
40
50
60
70
80
90
100
% span
ETB
MIXED
EXACT

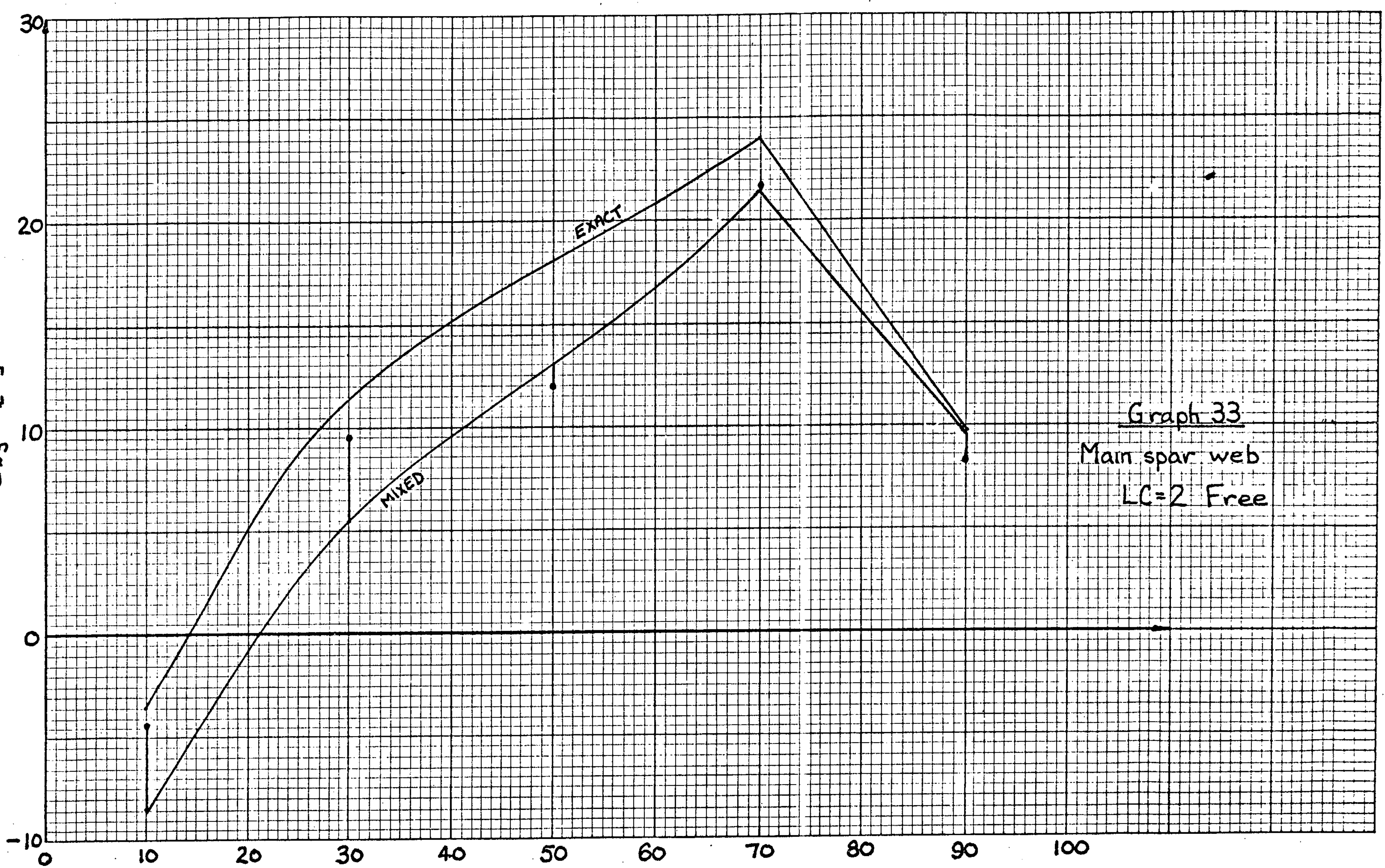

$\epsilon_{\bar{x}\bar{x}}$ [μ] +'ve
0
10
20
30
40
% span
EXACT
MIXED
Graph 29
Top skin panels
LC=2 Free
100

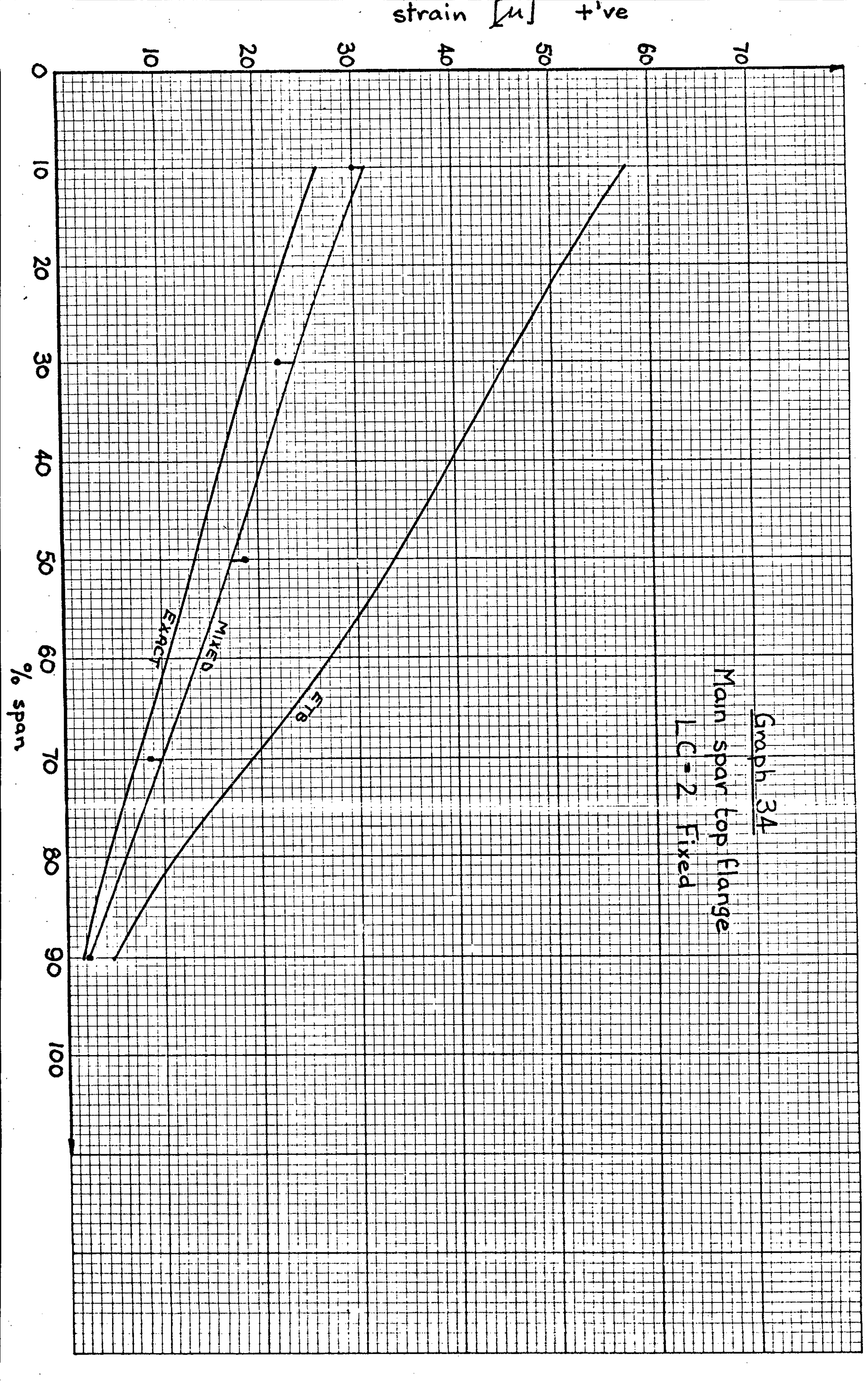


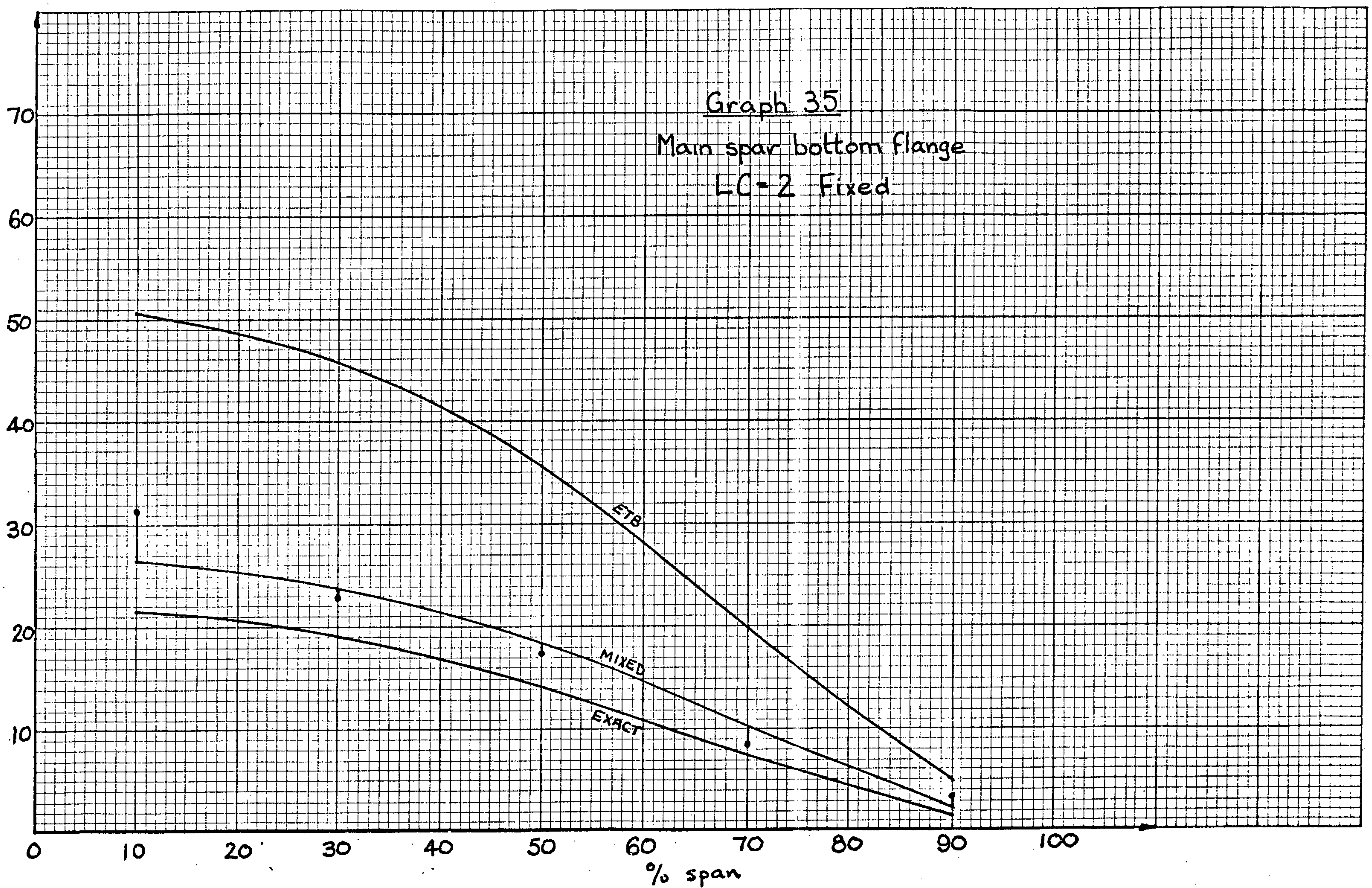
Graph 30
Bottom skin panels
LC=2 Free
MIXED
EXACT
0
10
20
30
40
50
60
70
80
90
100
% span
40
30
20
10


Graph 31
Top skin panels
LC=2 Free
MIXED
EXACT
$\epsilon_{\bar{x}\bar{y}}$ [µ] +'ve
% span
0
10
20
30
40
50
60
70
80
90
100

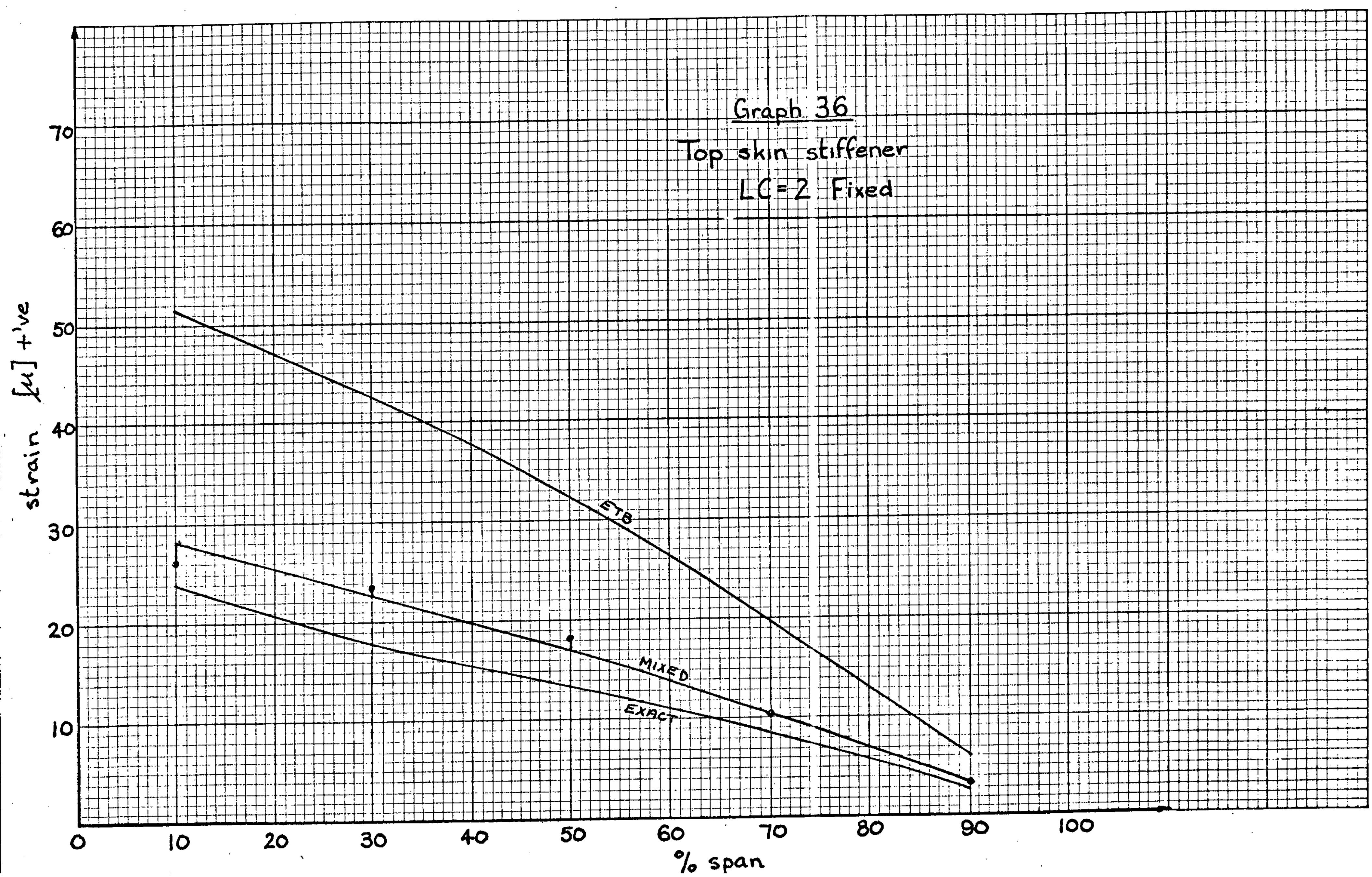

Graph 32
Bottom skin panels
LC=2 Free
MIXED
EXACT
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
% span

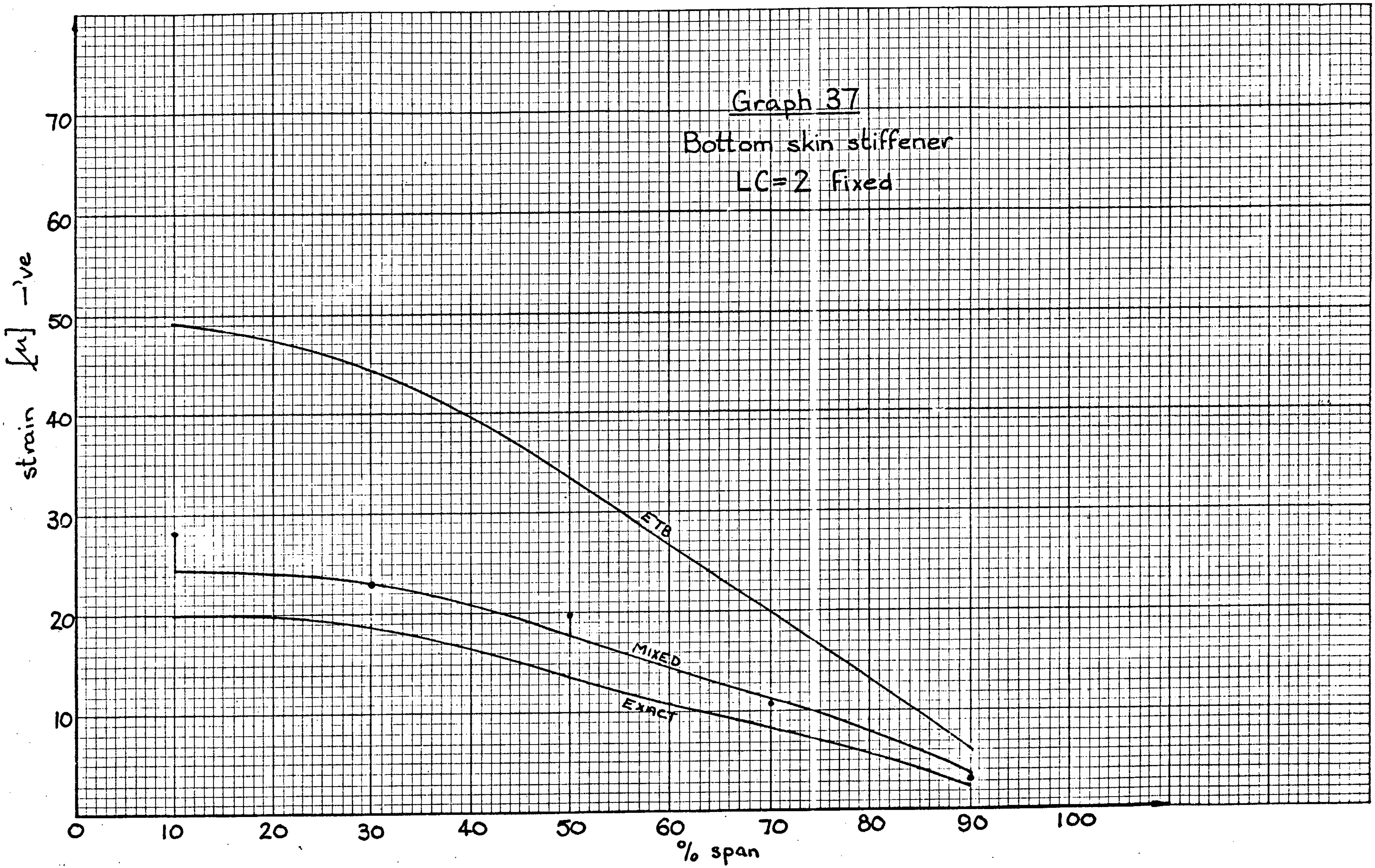

Graph 33
Main spar web
LC=2 Free
EXACT
MIXED
30
20
10
0
-10
0
10
20
30
40
50
60
70
80
90
100


Graph 34
Main spar top flange
LC-2 Fixed
strain [μ] +'ve
% span
EXACT
MIXED
ETB
0
10
20
30
40
50
60
70
80
90
100

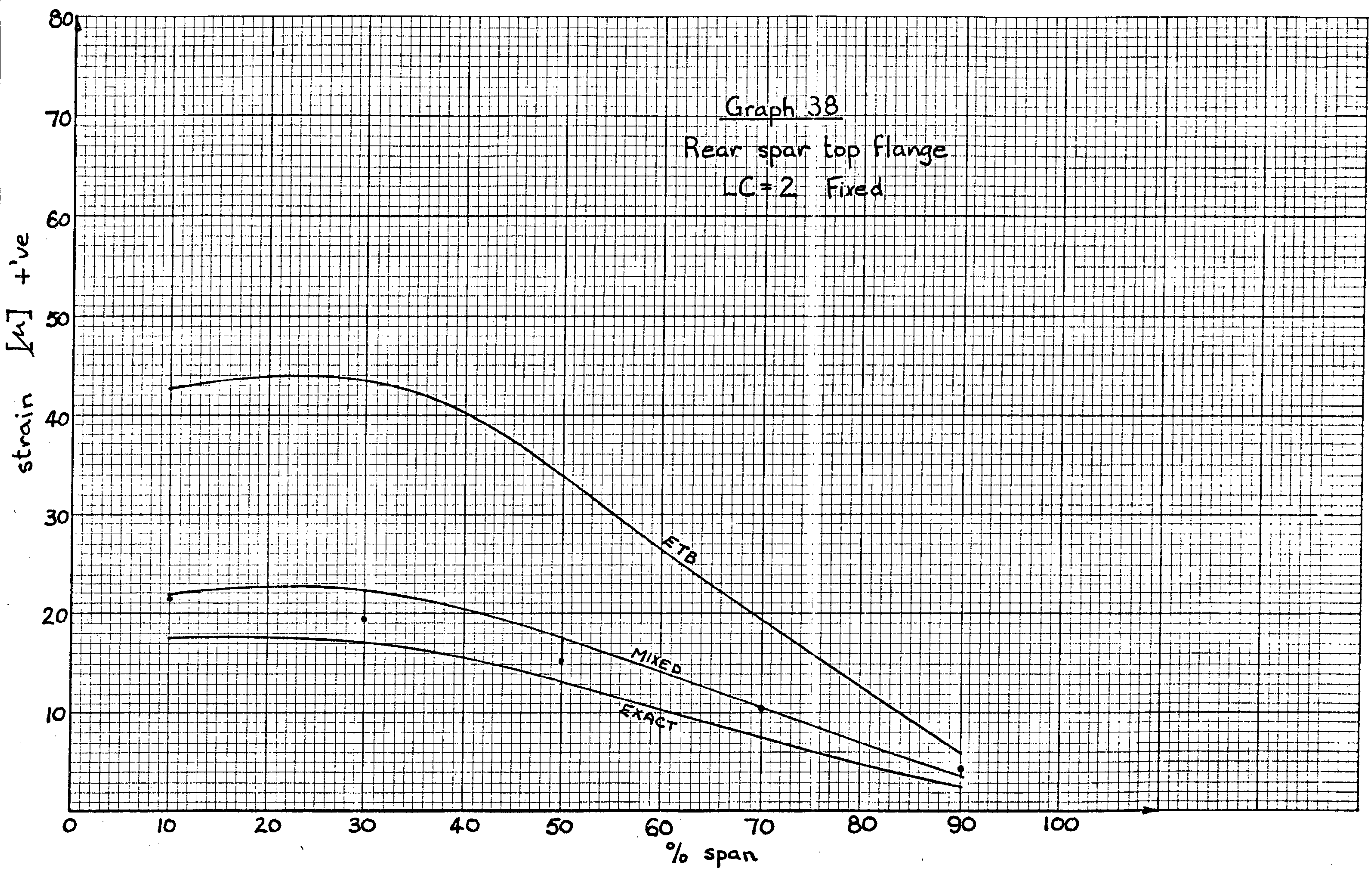


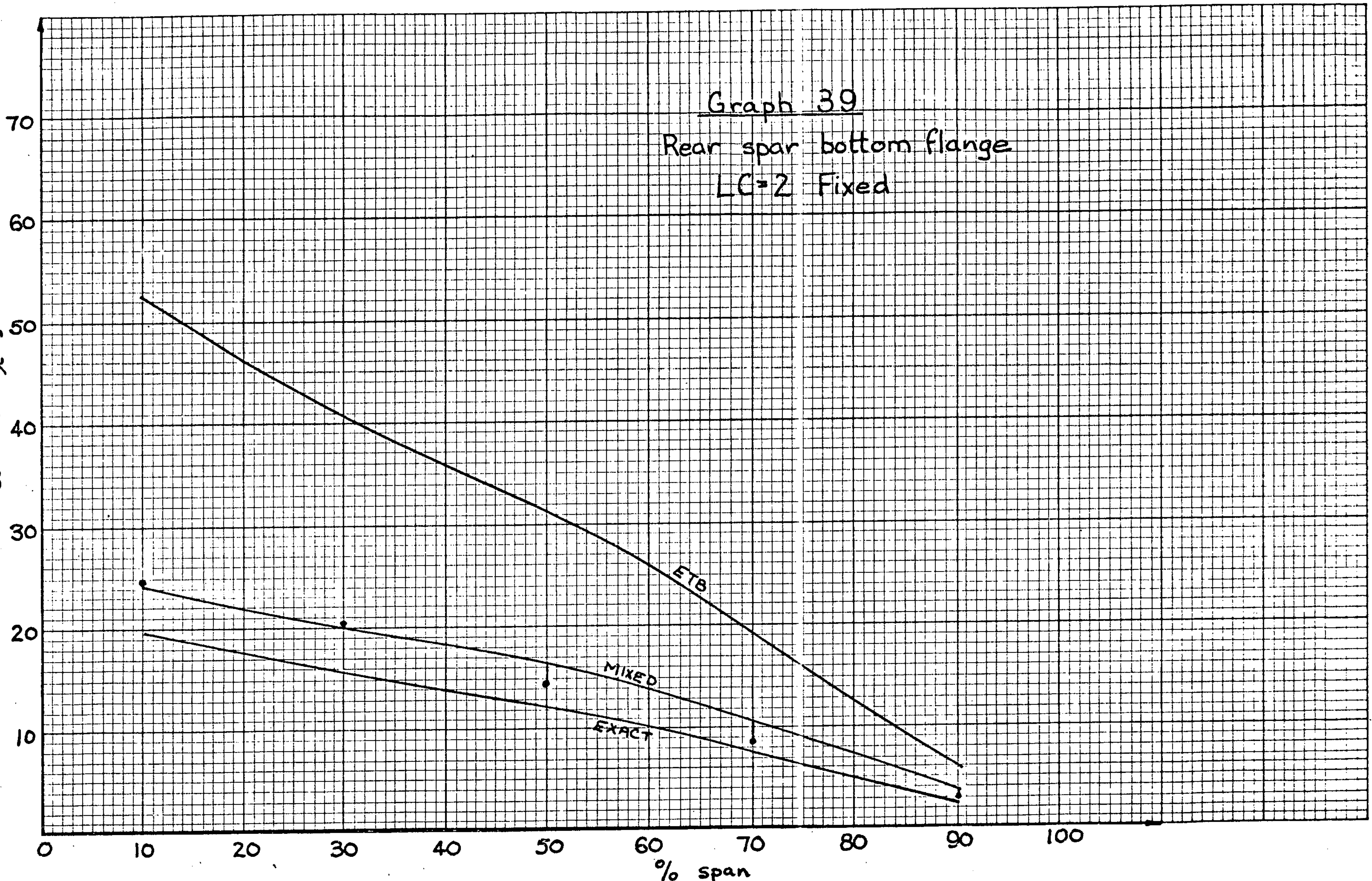
Graph 35
Main spar bottom flange
LC-2 Fixed
ETB
MIXED
EXACT
% span
0
10
20
30
40
50
60
70
80
90
100
70
60
50
40
30
20
10

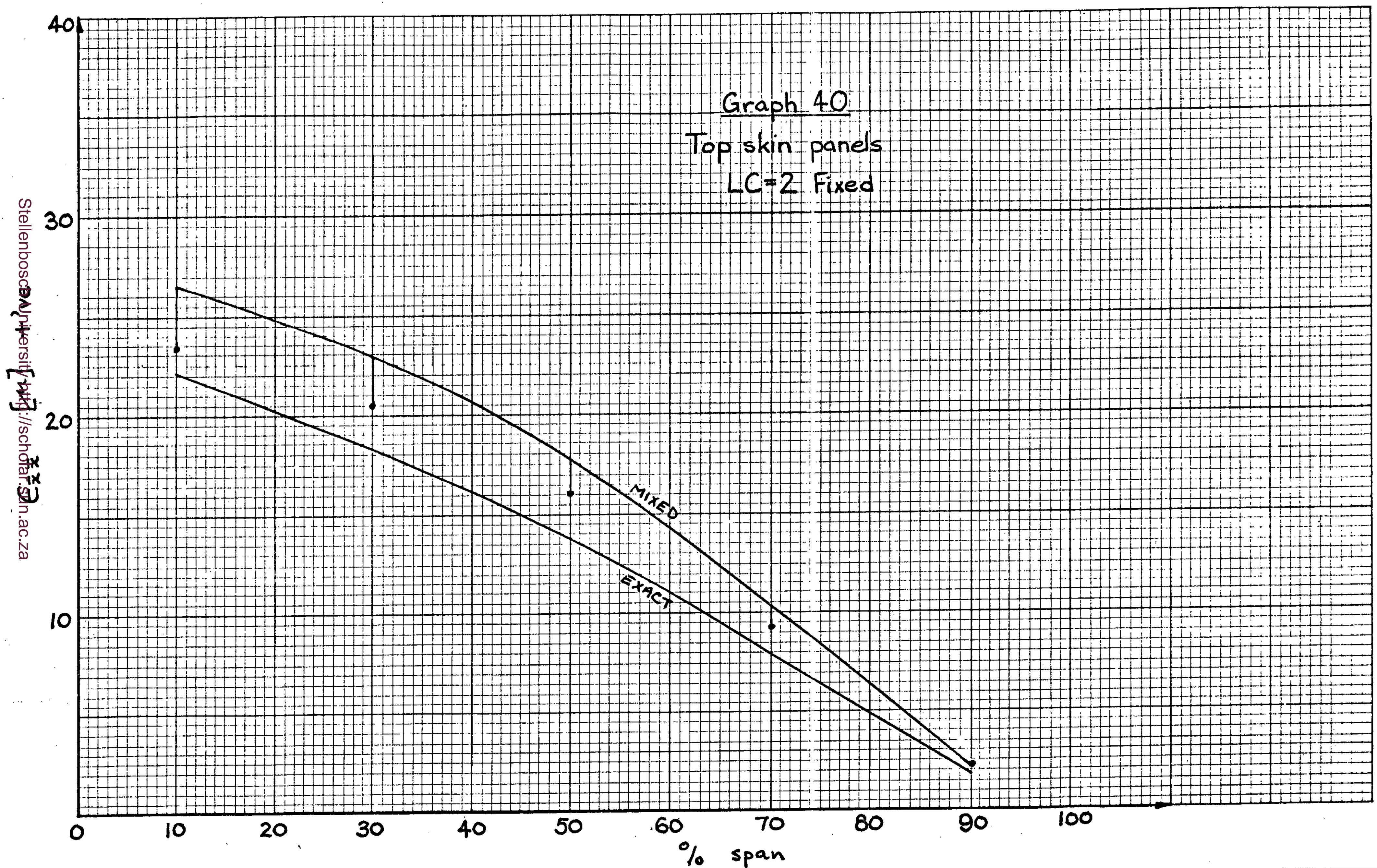

Graph 36
Top skin stiffener
LC=2 Fixed
strain [μ] +'ve
% span
ETB
MIXED
EXACT
0
10
20
30
40
50
60
70
80
90
100



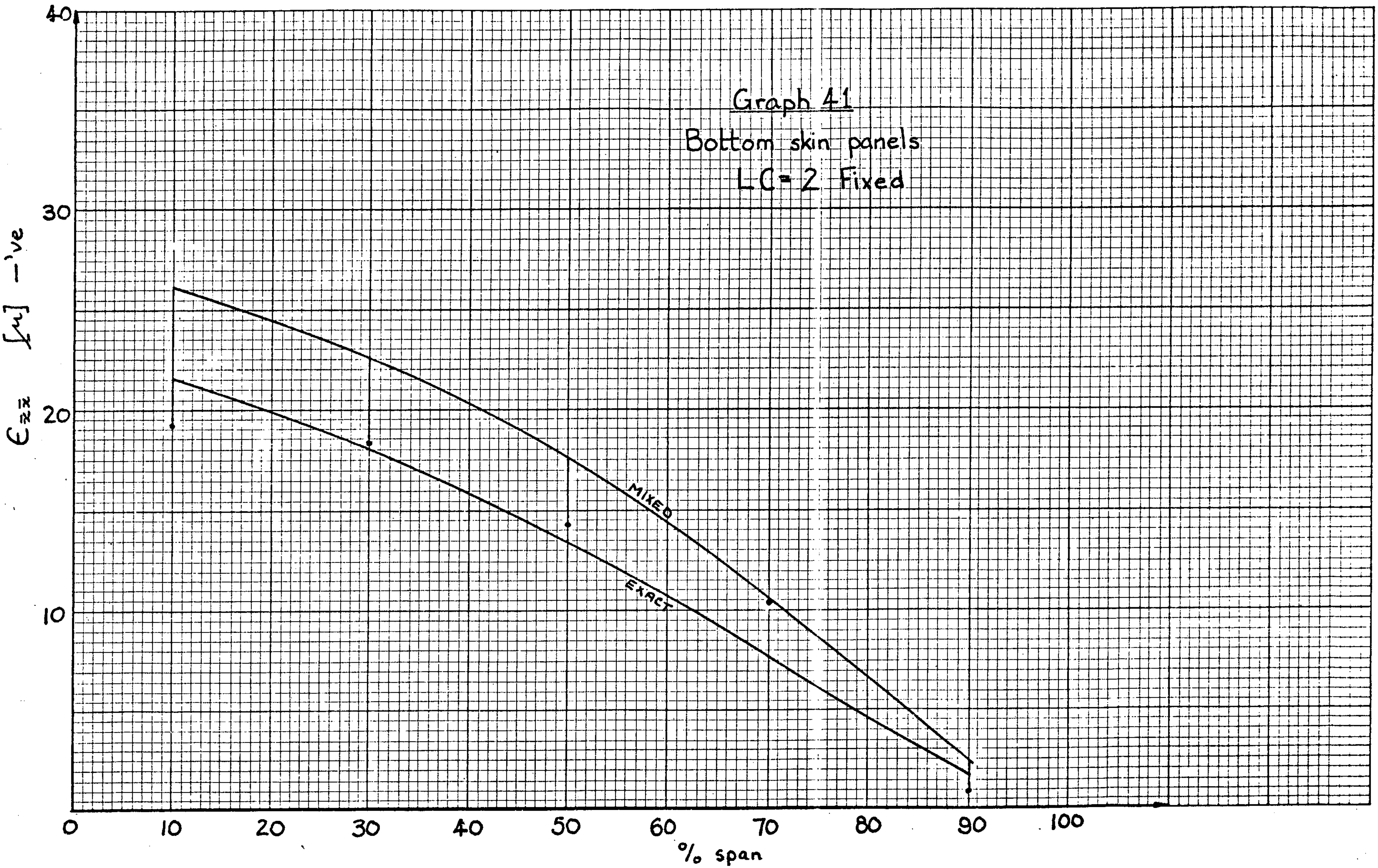
Graph 37
Bottom skin stiffener
LC=2 Fixed
strain [μ] -'ve
% span
ETB
MIXED
EXACT
0
10
20
30
40
50
60
70
80
90
100


Graph 38
Rear spar top flange
LC = 2 Fixed
strain [μ] +'ve
% span
ETB
MIXED
EXACT
0
10
20
30
40
50
60
70
80
90
100

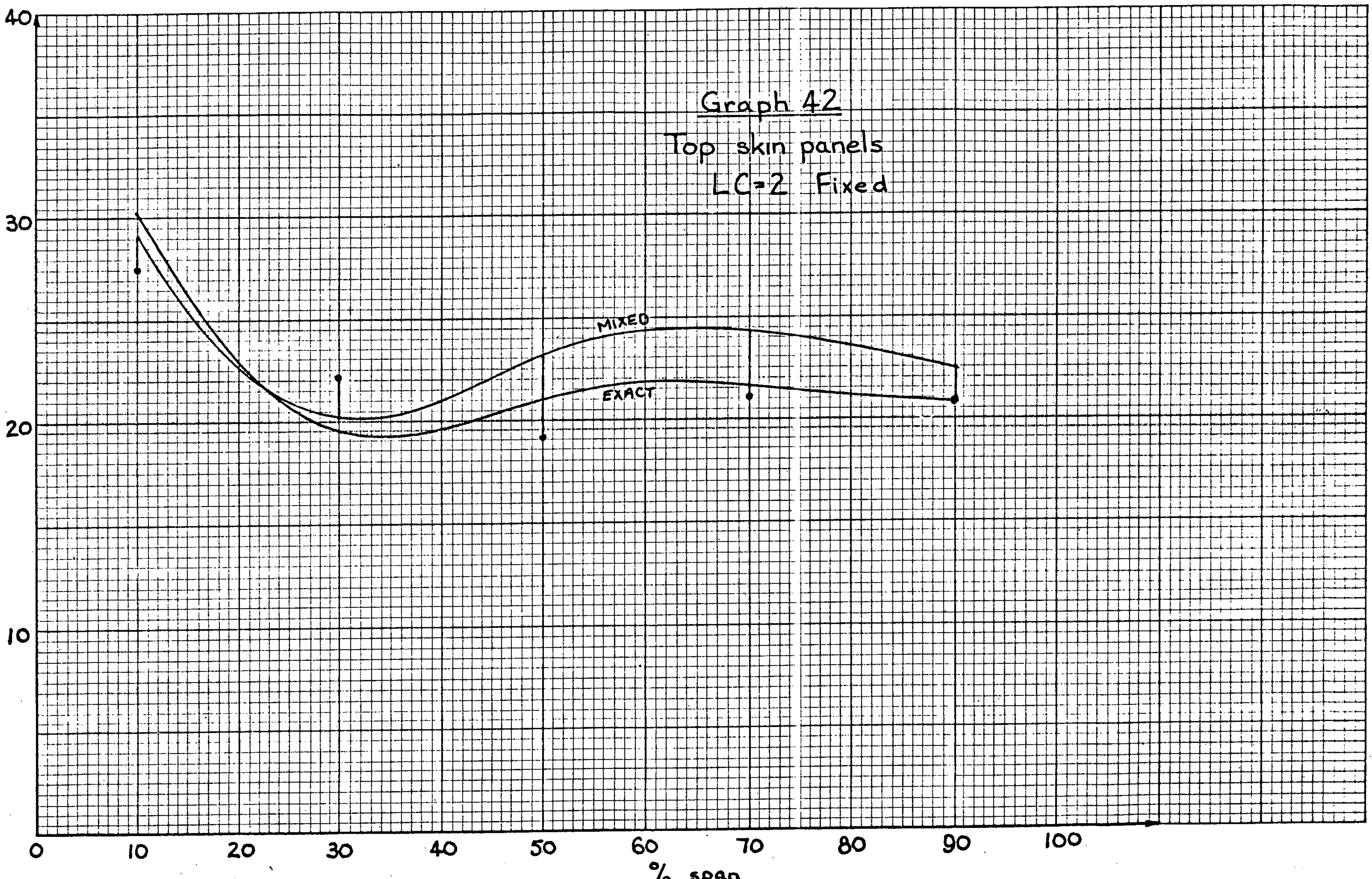

Graph 39
Rear spar bottom flange
LC=2 Fixed
ETB
MIXED
EXACT
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90
100
% span

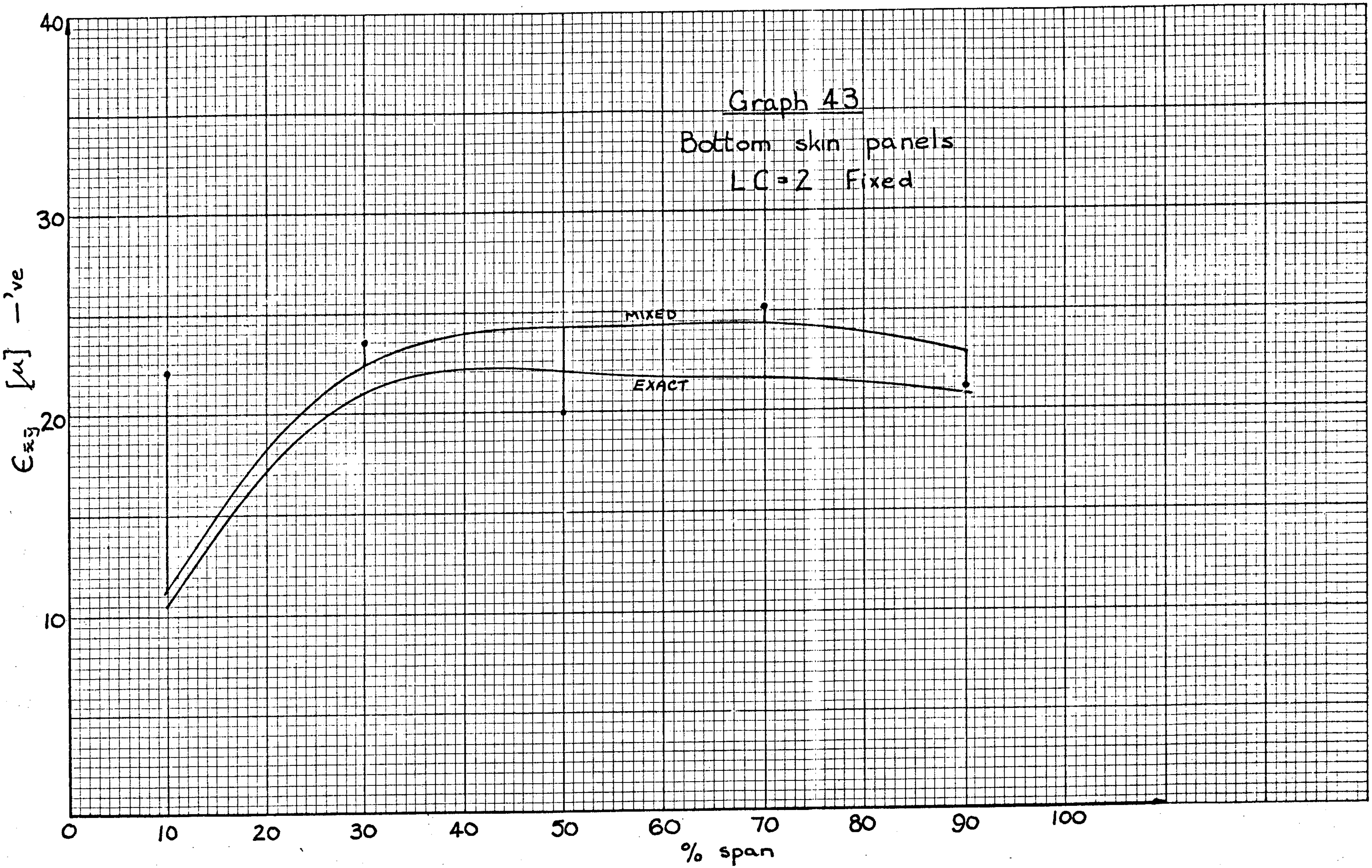


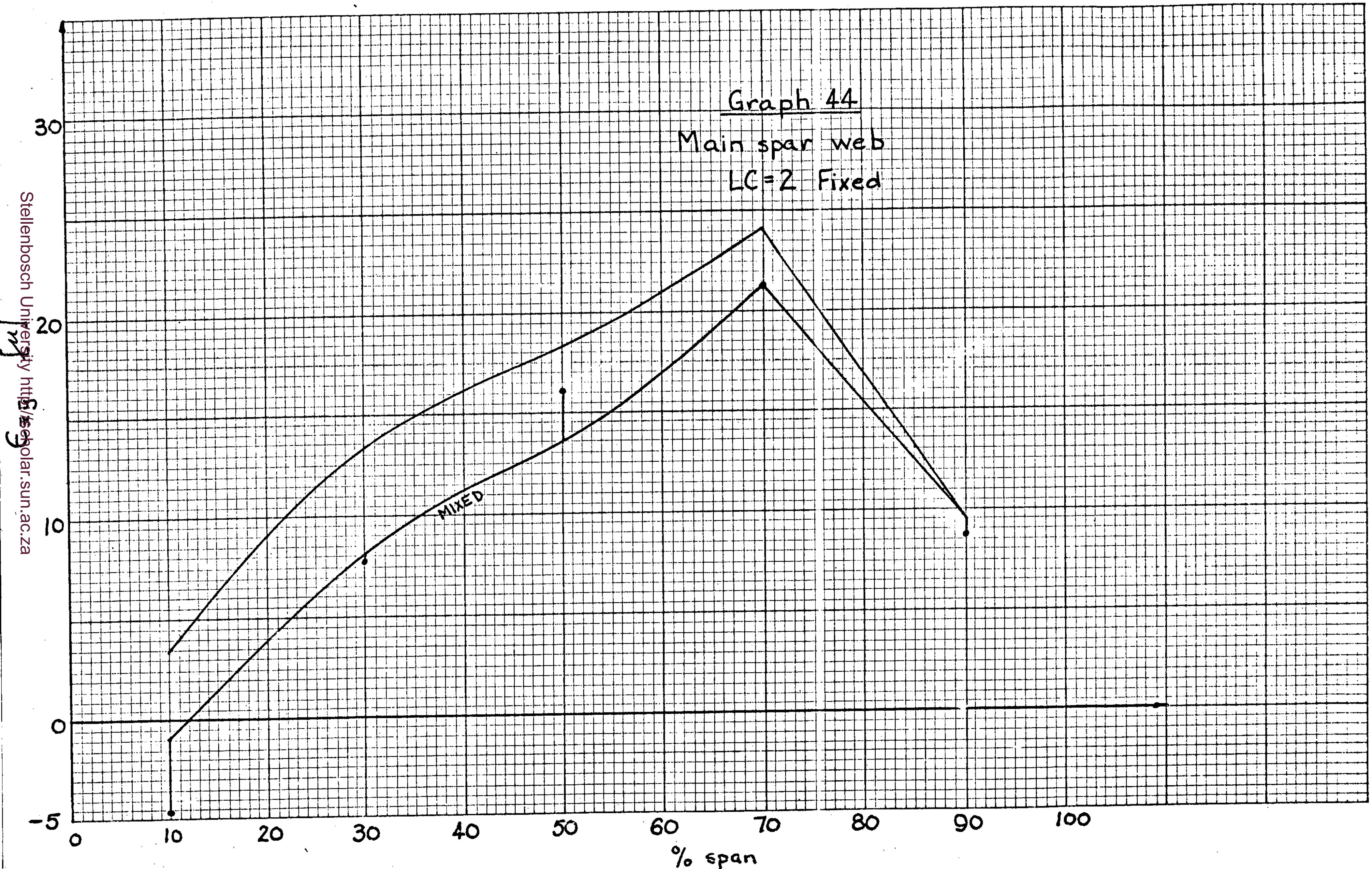
Graph 40
Top skin panels
LC=2 Fixed
MIXED
EXACT
% span
0
10
20
30
40
50
60
70
80
90
100
10
20
30
40



Graph 4.1
Bottom skin panels
LC = 2 Fixed
MIXED
EXACT
ϵ_{zz} [μ] –'ve
0
10
20
30
40
0
10
20
30
40
50
60
70
80
90
100
% span


Graph 42
Top skin panels
LC=2 Fixed
MIXED
EXACT
% span
0
10
20
30
40
50
60
70
80
90
100
10
20
30
40


Graph 43
Bottom skin panels
LC = 2 Fixed
ϵxy [μ] -'ve
% span
0
10
20
30
40
50
60
70
80
90
100
10
20
30
40
MIXED
EXACT



Graph 44
Main spar web
LC=2 Fixed
MIXED
30
20
10
0
-5
0
10
20
30
40
50
60
70
80
90
100
% span